Generalized Yang-Mills Fields and Gauge fields in $(A)dS_d$

E.D. Skvortsov

Lebedev Institute, Moscow

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Gravity as a Gauge Theory

$$g_{\mu\nu} \longrightarrow h^a_\mu,\ \varpi^{a,b}_\mu$$

Yang-Mills field $A_\mu dx^\mu = (P_a h^a_\mu + L_{a,b}\varpi^{a,b}_\mu)dx^\mu$

Algebra

$$\begin{array}{ll} [P_a, P_b] = \pm\lambda^2 L_{ab} & \lambda^2 > 0 \ \mathfrak{so}(d,1) \\ [L_{ab}, L_c] = L_a\eta_{bc} - L_b\eta_{ac} & \lambda^2 = 0 \ \mathfrak{iso}(d-1,1) \\ [L_{ab}, L_{cd}] = L_{ad}\eta_{bc} + \ldots & \lambda^2 < 0 \ \mathfrak{so}(d-1,2) \end{array}$$

Field strength $R = dA + [A, \wedge A] = P_a R^a + L_{a,b} R^{a,b}$

Torsion $R^a = dh^a + \varpi^{a,}{}_b \wedge h^b = 0$

Curvature $R^{a,b} = d\varpi^{a,b} + \varpi^{a,}{}_c \wedge \varpi^{c,b} \pm \lambda^2 h^a \wedge h^b = 0$

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Gravity as a Gauge Theory

$$d = 4$$

$$\frac{1}{\lambda^2}\int R^{a,b} \wedge R^{c,d} \epsilon_{abcd} \quad \sim \quad \int \sqrt{g}\left(R + \lambda^2\right) + \text{Gauss Bonnet}$$

$$d \geq 4$$

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Getting $(A)dS_d$ symmetry manifest

(MacDowell, Mansouri, Stelle, West, Vasiliev)

$$\Omega_\mu^{A,B} = -\Omega_\mu^{B,A}$$

$$A = 0, ..., d(\bullet) = a, \bullet$$

$$R^{A,B} = d\Omega^{A,B} + \Omega^{A,}{}_C \wedge \Omega^{C,B}$$

$$h^a = \lambda \Omega^{a,\bullet} \qquad R^a = \lambda R^{a,\bullet}$$

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Goldstone field	$V^A V^B \eta_{AB} = \pm 1$
Frame field	$H^A = D_\Omega V^A = dV^A + \Omega^{A,}{}_C V^C$
Spin-connection	$\Omega_L^{A,B} = \Omega^{A,B} \pm \lambda(H^A V^B - H^B V^A)$

Standard gauge $V^A = \delta^A_\bullet$

$$\frac{1}{\lambda^2} \int R^{A,B} \wedge R^{C,D} \epsilon_{ABCDE...UN} H^E ... H^U V^N$$

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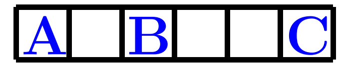
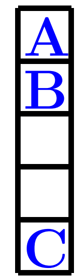
Tensors and Young Diagrams

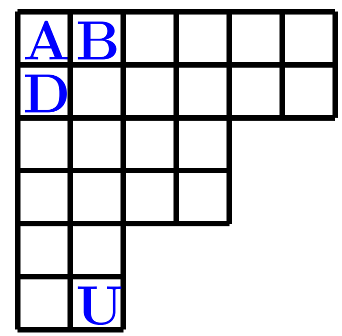
symmetric $T^{A \dots B \dots C} = T^{B \dots A \dots C}$



antisymmetric $T^{A,B,\dots,C} = -T^{B,A,\dots,C}$



mixed-symmetry $T^{AB \dots, D \dots, \; \dots \; , \dots U}$



T^A

T^{AB}

$T^{A,B} + T^{B,A} \equiv 0$

T^{ABC}

$T^{AB,C} + T^{BC,A} + T^{CA,B} \equiv 0$

$T^{A,B,C} + T^{B,A,C} \equiv 0$

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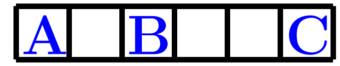

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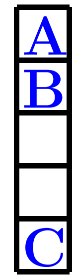

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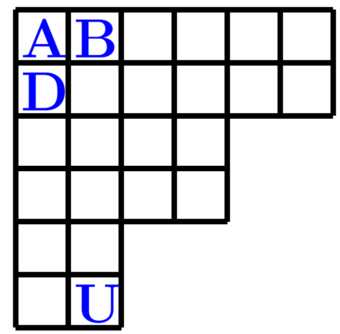
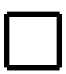
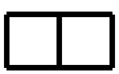
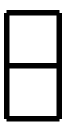
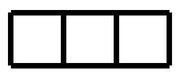
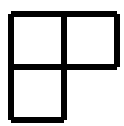


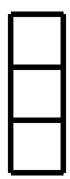
 T^{A}

 T^{AB}

 $T^{A,B} + T^{B,A} \equiv 0$

 T^{ABC}

 $T^{AB,C} + T^{BC,A} + T^{CA,B} \equiv 0$

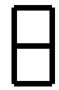
 $T^{A,B,C} + T^{B,A,C} \equiv 0$

Generalized Yang-Mills Connections

(Alkalaev, Shaynkman, Skvortsov, Vasiliev)

Yang-Mills Connection

$$W_{\mu}^{A,B} dx^{\mu}$$

one-form in adjoint= 

$$\Downarrow$$

Generalized Yang-Mills Connection

q-from in arbitrary module $\mathbb{A}$

$$W_{\mathbf{q}}^{\mathbf{A}} \equiv W_{\mu_1 \dots \mu_q}^{ABC \dots U} dx^{\mu_1} \wedge \dots \wedge dx^{\mu_q}$$

$\mathbb{A} =$

of $\mathfrak{so}(d,1)$ or $\mathfrak{so}(d-1,2)$

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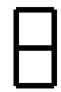
one-form in adjoint= 

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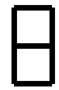
one-form in adjoint= 

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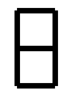
one-form in adjoint= 

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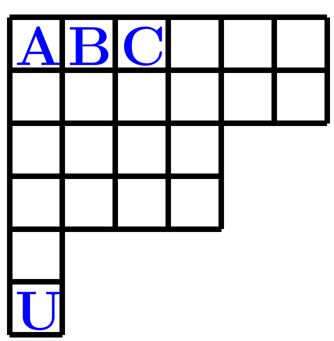
$\mathbf{A} =$ 

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Gauging with Generalized Connections

(Alkalaev, Shaynkman, Skvortsov, Vasiliev)

$$d\Omega^{A,B} + \Omega^{A,}{}_{C} \wedge \Omega^{C,B} = 0 \quad (A)dS_d$$

Covariant derivative

$$D_\Omega W_{\mathbf{q}}^{ABC...} = dW_{\mathbf{q}}^{ABC...} + \Omega^{A,}{}_{M} \wedge W_{\mathbf{q}}^{MBC...} + ...$$

$$D_\Omega{}^2 = 0$$

Bianchi Identities	$D_\Omega R^{\mathbf{A}}_{\mathbf{q+1}}$	$= 0$
Field Strength	$R^{\mathbf{A}}_{\mathbf{q+1}}$	$= D_\Omega W^{\mathbf{A}}_{\mathbf{q}}$
Gauge Transformations	$\delta W^{\mathbf{A}}_{\mathbf{q}}$	$= D_\Omega \xi^{\mathbf{A}}_{\mathbf{q-1}}$
Reducible Gauge Transformations	$\delta \xi^{\mathbf{A}}_{\mathbf{q-1}}$	$= D_\Omega \xi^{\mathbf{A}}_{\mathbf{q-2}}$

Manifest (anti)-de Sitter and gauge symmetry

What theory does $W^{\mathbf{A}}_{\mathbf{q}}$ describe?

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What theory does $W^{\mathbf{A}}_{\mathbf{q}}$ describe?

What should we expect?

Simplest gauge fields

massless spin-2

$\delta\phi_{\mu\nu} = D_\mu \xi_\nu + D_\nu \xi_\mu$

p.d.o.f.$=\pm 2, \pm \mathbf{1}, \mathbf{0}$

partially-massless spin-2
(Deser, Nepomechie)

$\delta\phi_{\mu\nu} = D_\mu D_\nu \xi + \lambda^2 g_{\mu\nu}\xi$

p.d.o.f.$=\pm 2, \pm 1, 0$

massless spin-s
(Fronsdal,Vasiliev)

$\delta\phi_{\mu_1\ldots\mu_s} = D_{\mu_1}\xi_{\mu_2\ldots\mu_s} +$ **permutations**

p.d.o.f.$=\pm s, \pm(\mathrm{s}-1), \ldots, 0$

partially-massless spin-s
(Deser, Waldron, Higuchi)
(Witten, Zinoviev)

$\delta\phi_{\mu_1\ldots\mu_s} = D_{\mu_1}\ldots D_{\mu_t}\xi_{\mu_{t+1}\ldots\mu_s} + \ldots$

p.d.o.f.$=\pm s, \ldots, \pm(s-t), \pm(\mathrm{s}-\mathrm{t}-1), \ldots, 0$

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p.d.o.f.$= \pm 2, \pm 1, 0$

massless spin-s
(Fronsdal,Vasiliev)

$\delta\phi_{\mu_1 \ldots \mu_s} = D_{\mu_1} \xi_{\mu_2 \ldots \mu_s} +$ **permutations**
p.d.o.f.$= \pm s, \pm(s-1), \ldots, 0$

partially-massless spin-s
(Deser, Waldron, Higuchi)
(Witten, Zinoviev)

$\delta\phi_{\mu_1 \ldots \mu_s} = D_{\mu_1} \ldots D_{\mu_t} \xi_{\mu_{t+1} \ldots \mu_s} + \ldots$
p.d.o.f.$= \pm s, \ldots, \pm(s-t), \pm(s-t-1), \ldots, 0$

What should we expect?

Simplest gauge fields

massless spin-2

$\delta\phi_{\mu\nu} = D_\mu \xi_\nu + D_\nu \xi_\mu$

p.d.o.f.=$\pm 2, \pm\mathbf{1}, \mathbf{0}$

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p.d.o.f.$=\pm s, \ldots, \pm(s-t), \pm(\mathbf{s-t-1}), \ldots, \mathbf{0}$

Higher-Spin fields

String theory $\xrightarrow{\alpha' \to \infty}$ **higher-spin fields**

(Gross, Klebanov, Sundborg, Sezgin, Bengtsson, Sagnotti, Francia, Bonelly)

Breaking HS symmetries

$$|\phi\rangle = \phi_{\mu_1 \dots \mu_s}(x) \alpha^{\mu_1}_{-1} \dots \alpha^{\mu_s}_{-1} |0\rangle$$

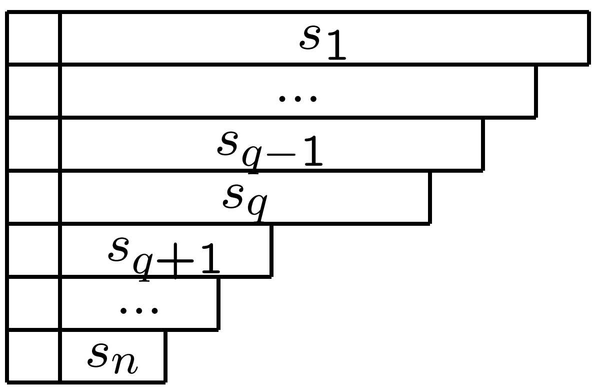
$\alpha^{\mu}_{-1} \dots . \alpha^{\nu}_{-k}$ - mixed-symmetry fields

What should we expect?

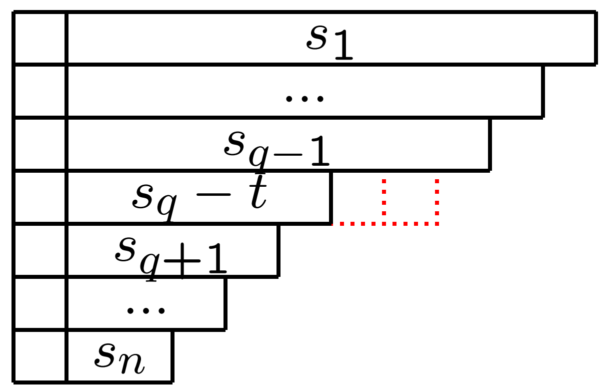
General case

(Alkalaev,Skvortsov,Metsaev,Vasiliev)

Spin of the field $\mathbf{s}_0$



Spin of the gauge parameter $\mathbf{s}_1$



$$\delta\phi^{\mathbf{s}_0} = \overbrace{D...D}^{t}\,\xi^{\mathbf{s}_1} + ...$$

Gauge Fields in $(A)dS_d \Longleftrightarrow (S, q, t)$

The way it works

Simplest cases

$(A)dS_d$-covariant frame-like $\xrightarrow{V^A}$ Lorentz-covariant frame-like $\underset{h_\mu^a}{\xrightarrow{h_a^\mu}}$ Lorentz-covariant metric-like

$\mathbf{A} =$ $, \; q = 1$

$W_{\mathbf{1}}^{A,B}$

$\delta W_{\mathbf{1}}^{A,B} = D_\Omega \xi_{\mathbf{0}}^{A,B}$

$\phi_{\mu\nu} = W_\mu^{c,\bullet} h_{c\nu} + W_\nu^{c,\bullet} h_{c\mu}$

$\delta\phi_{\mu\nu} = D_\mu \xi_\nu + D_\nu \xi_\mu$

$\xi_\mu = \xi^{c,\bullet} h_{c\nu}$

$\mathbf{A} =$ $, \; q = 1$

$\phi_{\mu_1 \ldots \mu_s} = W_{\mu_1}^{a..c,\bullet\ldots\bullet} h_{a\mu_2} \ldots h_{c\mu_s} + \ldots$

$\delta\phi_{\mu_1 \ldots \mu_s} = D_{\mu_1} \ldots D_{\mu_t} \xi_{\mu_{t+1} \ldots \mu_s} + \ldots$

$\xi_{\mu_1 \ldots \mu_{s-t}} = \xi^{a..c\bullet\ldots\bullet,\bullet\ldots\bullet} h_{a\mu_1} \ldots h_{c\mu_{s-t}}$

The way it works

Simplest cases

$(A)dS_d$-covariant frame-like $\xrightarrow{V^A}$ Lorentz-covariant frame-like $\underset{h^a_\mu}{\xrightarrow{h^\mu_a}}$ Lorentz-covariant metric-like

$$\mathbf{A} = \begin{array}{|c|}\hline \\ \hline \\ \hline \end{array} \;,\; q = 1$$

$$W_{\mathbf{1}}^{A,B}$$
$$\delta W_{\mathbf{1}}^{A,B} = D_\Omega \xi_{\mathbf{0}}^{A,B}$$

$$\phi_{\mu\nu} = W_\mu^{c,\bullet} h_{c\nu} + W_\nu^{c,\bullet} h_{c\mu}$$
$$\delta\phi_{\mu\nu} = D_\mu \xi_\nu + D_\nu \xi_\mu$$
$$\xi_\mu = \xi^{c,\bullet} h_{c\nu}$$

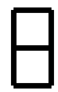
$$\mathrm{A} = \begin{array}{|c|c|}\hline \multicolumn{2}{|c|}{s-1} \\ \hline s-t & \\ \cline{1-1} \end{array} ,\; q = 1$$

$$\phi_{\mu_1\ldots\mu_s} = W_{\mu_1}^{a..c,\bullet\ldots\bullet} h_{a\mu_2} \ldots h_{c\mu_s} + \ldots$$
$$\delta\phi_{\mu_1\ldots\mu_s} = D_{\mu_1} \ldots D_{\mu_t} \xi_{\mu_{t+1}\ldots\mu_s} + \ldots$$
$$\xi_{\mu_1\ldots\mu_{s-t}} = \xi^{a..c\bullet\ldots\bullet,\bullet\ldots\bullet} h_{a\mu_1} \ldots h_{c\mu_{s-t}}$$

The way it works

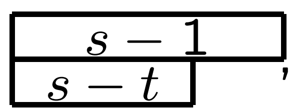
Simplest cases

$(A)dS_d$-covariant frame-like	$\xrightarrow{V^A}$	Lorentz-covariant frame-like	$\underset{h^a_\mu}{\xrightarrow{h^\mu_a}}$	Lorentz-covariant metric-like

$\mathbf{A} =$  $, \; q = 1$

$$W_{\mathbf{1}}^{A,B}$$
$$\delta W_{\mathbf{1}}^{A,B} = D_\Omega \xi_{\mathbf{0}}^{A,B}$$

$$\phi_{\mu\nu} = W_\mu^{c,\bullet} h_{c\nu} + W_\nu^{c,\bullet} h_{c\mu}$$
$$\delta\phi_{\mu\nu} = D_\mu \xi_\nu + D_\nu \xi_\mu$$
$$\xi_\mu = \xi^{c,\bullet} h_{c\nu}$$

$\mathbf{A} =$ 

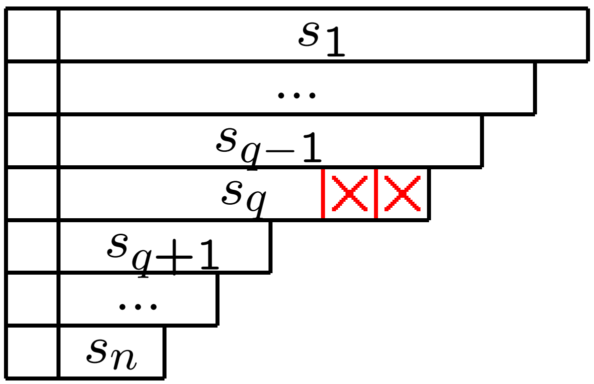
$, \; q = 1$

$$\phi_{\mu_1 \ldots \mu_s} = W_{\mu_1}^{a..c,\bullet \ldots \bullet} h_{a\mu_2} \ldots h_{c\mu_s} + \ldots$$
$$\delta\phi_{\mu_1 \ldots \mu_s} = D_{\mu_1} \ldots D_{\mu_t} \xi_{\mu_{t+1} \ldots \mu_s} + \ldots$$
$$\xi_{\mu_1 \ldots \mu_{s-t}} = \xi^{a..c\bullet \ldots \bullet, \bullet \ldots \bullet} h_{a\mu_1} \ldots h_{c\mu_{s-t}}$$

Gauge Fields vs. Gauge Connections

Lorentz metric-like

$\mathfrak{so}(d-1,1)$

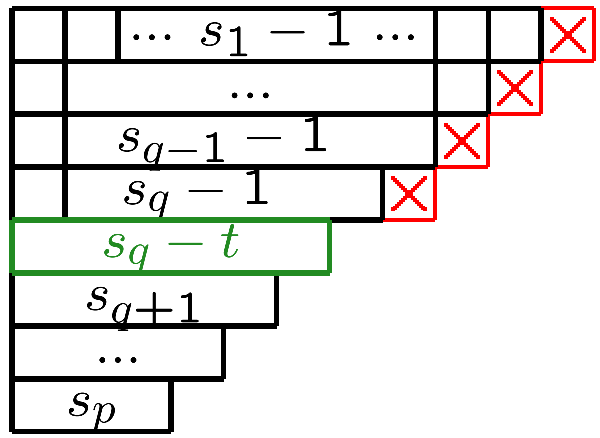


$$\delta\phi^{\mathbf{S}_0} = \overbrace{D\ldots D}^{t}\,\xi^{\mathbf{S}_1} + \ldots$$

extremely complicated

Connection

$\mathfrak{so}(d,1)$ or $\mathfrak{so}(d-1,2)$



$$\delta W^{\mathbf{A}}_{\mathbf{q}} = D_\Omega \xi^{\mathbf{A}}_{\mathbf{q-1}}$$

$$R^{\mathbf{A}}_{\mathbf{q+1}} = D_\Omega W^{\mathbf{A}}_{\mathbf{q}}$$

Great simplifications

Conclusions

$(A)dS_d$-Gauge Fields

$$\updownarrow$$

Yang-Mills Connections

- **Manifest gauge invariance**
- **Manifest $(A)dS_d$ covariance**
- **Manifest general covariance**
- **Nonlinear theory??** $\longrightarrow$ **String theory**