Gauge invariant approach

to conformal currents

and

shadow fields

R.R. Metsaev

Lebedev Institute

Plan

1) gauge invariant formulation

of conformal currents and shadow fields

2) AdS/CFT correspondence for

conformal currents and shadows

and massless AdS fields

Spin-1 conformal current
Standard approach

$$T^a \quad - \quad conformal \quad current$$

$$\partial^a T^a = 0$$

Conformal dimension

$$\Delta = d - 1$$

Spin-1 conformal current. Gauge inv. approach

$$\phi^a_{cur} \qquad \phi_{cur}$$

$$\mathbf{T^a = \phi^a_{cur} + \partial^a \phi_{cur}}$$

$$\partial^a T^a = 0$$

$$\mathbf{\partial^a \phi^a_{cur} + \Box \phi_{cur} = 0}$$

$$\delta \phi^a_{cur} = \partial^a \xi$$

$$\delta \phi_{cur} = -\xi$$

Conformal dimensions

$$\Delta(T^a) = d - 1$$

$$T^a = \phi^a_{cur} + \partial^a \phi_{cur}$$

$$\boldsymbol{\Delta(\phi^{a}_{cur}) = d - 1}$$

$$\boldsymbol{\Delta(\phi_{cur}) = d - 2}$$

Shadow spin-1 field: Definition

$\mathbf{T^a}$ $\quad$ *current*

$\mathbf{\Phi^a}$ $\quad$ *is shadow field* **IF**

$$\mathcal{L} = \mathbf{\Phi^a T^a}$$

is invariant w.r.t conformal

symmetries so(d,2)

$$\delta\Phi^a = \partial^a \xi$$

Spin-1 shadow field. Gauge inv. approach

$$\phi^{\mathbf{a}}_{\mathbf{sh}} \qquad\qquad \phi_{\mathbf{sh}}$$

$$\partial^{\mathbf{a}}\phi^{\mathbf{a}}_{\mathbf{sh}} + \phi_{\mathbf{sh}} = \mathbf{0}$$

$$\delta\phi^a_{sh} = \partial^a \xi_{sh}$$

$$\delta\phi_{sh} = -\Box\xi_{sh}$$

Conformal dims. Shadows

$$\Delta(\phi^a) = 1$$

$$\partial^a \phi^a_{sh} + \phi_{sh} = 0$$

$$\boldsymbol{\Delta(\phi^a_{sh}) = 1}$$

$$\boldsymbol{\Delta(\phi_{sh}) = 2}$$

Step 2

One needs to prove that

gauge invariant approach respects

conformal symmetries

of $so(d,2)$ algebra

Conformal algebra $so(d, 2)$

$\mathbf{P^a}$ translations

$\mathbf{J^{ab}}$ Lorentz rotations

$\mathbf{D}$ Dilatation

$\mathbf{K^a}$ conformal boosts

All that remains is to respect conformal boost symmetries

$$K^a$$

$$K^a = \tilde{K}^a + \mathbf{R^a}$$

$$\tilde{K}^a \equiv -\frac{1}{2}x^2\partial^a + x^a D + M^{ab}x^b$$

we have to find operator

$$R^a$$

Currents

$$R^a \phi^b_{cur} = \eta^{ab} \phi_{cur}$$

$$R^a \phi_{cur} = 0$$

Shadows

$$R^a \phi^b_{sh} = 0$$

$$R^a \phi_{sh} = \phi^a_{sh}$$

Spin-2 conformal current (energy-momentum tensor) Standard approach

$$\mathbf{T^{ab}} \qquad - \, spin \; 2 \; conformal \;\; current$$

$$\partial^{\mathbf{a}} \mathbf{T^{ab} = 0}$$

$$\mathbf{T^{aa} = 0}$$

Conformal dimension

$$\mathbf{\Delta = d}$$

Spin-2 current. Gauge inv. approach

Fields	$Conf.dim$
$\phi_{\mathbf{cur}}^{\mathbf{ab}}$	d
$\phi_{\mathbf{cur}}^{\mathbf{a}}$	$d-1$
$\phi_{\mathbf{cur}}$	$d-2$

Spin 2. Currents. Differential constraints

$$\partial^b \phi^{ab}_{cur} + \partial^a \phi^{bb}_{cur} + \Box \phi^a_{cur} = 0$$

$$\partial^a \phi^a_{cur} + \phi^{aa}_{cur} + \Box \phi_{cur} = 0$$

ϕ^a_{cur}, ϕ_{cur} can be gauged away

$$\partial^b \phi^{ab}_{cur} = 0$$

$$\phi^{aa}_{cur} = 0$$

Spin 2. Currents. Gauge transformations

$$\delta\phi^{ab}_{cur} = \partial^a \xi^b_{cur} + \partial^b \xi^a_{cur} + \eta^{ab} \Box \xi_{cur}$$

$$\delta\phi^{a}_{cur} = \partial^a \xi_{cur} + \xi^a_{cur}$$

$$\delta\phi_{cur} = \xi_{cur}$$

ϕ^a_{cur}, ϕ_{cur} Stueckelberg fields

Spin 2. Currents. Gauge fields and energy-momentum tensor

$$T^{ab} = \phi^{ab}_{cur} + \partial^a \phi^b_{cur} + \partial^b \phi^a_{cur}$$

$$+ \partial^a \partial^b \phi_{cur} + \eta^{ab} \Box \phi_{cur}$$

1) T^{ab} is gauge invariant

2) $\partial^a T^{ab} = 0\,, \qquad T^{aa} = 0$
amount to differential constraints

for ϕ^{ab}_{cur}, ϕ^a_{cur}, ϕ_{cur}.

Spin 2. Shadows

Fields	$Conf.dim$
$\phi^{\mathbf{ab}}_{\mathbf{sh}}$	0
$\phi^{\mathbf{a}}_{\mathbf{sh}}$	1
$\phi_{\mathbf{sh}}$	2

Spin 2. Shadows differential constraints

$$\partial^b \phi_{sh}^{ab} + \partial^a \phi_{sh}^{bb} + \phi_{sh}^{a} = 0$$

$$\partial^a \phi_{sh}^{a} + \Box \phi_{sh}^{aa} + \phi_{sh} = 0$$

ϕ_{sh}^{aa} can be gauged away

$$\partial^b \phi_{sh}^{ab} + \phi_{sh}^{a} = 0$$

$$\partial^a \phi_{sh}^{a} + \phi_{sh} = 0$$

Spin 2. Shadows gauge transformations

$$\delta\phi^{ab}_{sh} = \partial^a \xi^b_{sh} + \partial^b \xi^a_{sh} + \eta^{ab}\xi_{sh}$$

$$\delta\phi^{a}_{sh} = \partial^a \xi_{sh} + \Box \xi^a_{sh}$$

$$\delta\phi_{sh} = \Box \xi_{sh}$$

ϕ^{aa}_{sh} Stueckelberg field

ϕ^{a}_{sh}, ϕ_{sh} are not Stueckelberg fields

Generalization to

arbitrary spin currents and shadows

is straightforward by using

double-traceless

Fronsdal fields

Statement

Requiring

1) gauge invariance

2) conformal invariance so(d,2)

we obtain solution to

differential constraints

and operator R^a

AdS/CFT

Our currents and shadows

correspond to

bulk AdS fields taken

in modified de Donder gauge

summary of our study of AdS/CFT

1) Bulk fields are taken in
modified de-Donder gauge

2) **de Donder gauge** leads to

decoupled equations of motion
with on-shell leftover gauge symmetries

3) normalizable solutions $\longrightarrow$ currents

no-normalizable solutions $\longrightarrow$ shadows

4) leftover **gauge symmetries of bulk fields** correspond to **gauge symmetries of boundary currents and shadow fields**,

5) **de Donder gauge** for bulk fields corresponds to **differential constraints** for boundary conformal currents and shadows

AdS/CFT

$$AdS_{d+1} \quad space$$

$$ds^2 = \frac{1}{z^2}(dx^a dx^a + dzdz)$$

bulk $so(d,1) \rightarrow$ **boundary** $so(d-1,1)$

$$\phi^A \quad = \quad \phi^a \oplus \phi$$

AdS. Spin 1

$$D^A F^{AB} = 0$$

$$F^{AB} = D^A \phi^B - D^B \phi^A$$

$$\phi^{\mathbf{A}} \quad = \quad \phi^{\mathbf{a}} \quad \oplus \quad \phi$$

(in radial gauge, $\phi = \mathbf{0}$)

standard Lorentz gauge

$$D^A \phi^A = 0$$

leads to

coupled equations

$$(\Box + \partial_z^2 - m^2)\phi^{\mathbf{a}} + \partial^a \phi = 0$$

$$(\Box + \partial_z^2 - m^2)\phi + \partial^a \phi^{\mathbf{a}} = 0$$

Modified Lorentz gauge

$$D^A \phi^A + 2\phi = 0$$

RRM, 1999

Polchinski and Strassler 2001

gives

Decoupled equations

Decoupled equations

$$(\Box + \partial_z^2 - m_1^2)\phi^a = 0$$

$$(\Box + \partial_z^2 - m_0^2)\phi = 0$$

$$m_1^2 = \frac{1}{z^2}(\nu_1^2 - \frac{1}{4})$$

$$m_0^2 = \frac{1}{z^2}(\nu_0^2 - \frac{1}{4})$$

$$\phi^a(x,z) = \mathbf{U}_{\nu_1}\ \phi^a_{cur}(x)$$

$$\phi(x,z) = \mathbf{U}_{\nu_0}\ \phi_{cur}(x)$$

$$\mathbf{U}_\nu \equiv \sqrt{z}\mathbf{J}_\nu(\mathbf{z}\sqrt{\Box})$$

Bessel

modified Lorentz gauge

$$\partial^{\mathbf{a}}\phi^{\mathbf{a}} + \left(\partial_{\mathbf{z}} + \frac{a}{z}\right)\phi = 0$$

$$\partial^{\mathbf{a}}\phi^{\mathbf{a}} + \left(\partial_{\mathbf{z}} + \frac{a}{z}\right)\phi$$

$$= U_{\nu_1}(\partial^{\mathbf{a}}\phi^{\mathbf{a}}_{\mathbf{cur}} + \Box\phi_{\mathbf{cur}})$$

“technical” problem with standard cov. gauges, Lorentz, de Donder

1) Coupled equations

2) For spin 2, 3, 4, ...

solutions are expressible

in terms of **Heun functions**

Little is known about **Heun functions**

asymptotic behavior ???

recurrent relations ???

Spin 2: modified de Donder gauge

$$D^B h^{AB} - \frac{1}{2} D^A h + 2h^{zA} - \eta^{zA} h = 0$$

$$h \equiv h^{AA}$$

leads to **decoupled** equations

$$h^{AB} = h^{ab} \oplus h^{za} \oplus h^{zz}$$

$$T^{ab} = \phi^{ab}_{cur} + \partial^a \phi^b_{cur} + \partial^b \phi^a_{cur}$$

$$+ \ \partial^a \partial^b \phi_{cur} + \eta^{ab} \Box \phi_{cur}$$

Breaking conformal symmetry

$$\Box \rightarrow \mathbf{m^2}$$

$$\phi^{\mathbf{a}}_{\mathbf{cur}} \rightarrow \frac{1}{m} \phi^{\mathbf{a}} \qquad \phi_{\mathbf{cur}} \rightarrow \frac{1}{m^2} \phi^{\mathbf{a}}$$

$$T^{ab} = \phi^{\mathbf{ab}} + \frac{1}{m} (\partial^{\mathbf{a}} \phi^{\mathbf{b}} + \partial^{\mathbf{b}} \phi^{\mathbf{a}})$$

$$+ \ \frac{\partial^a \partial^b}{m^2} \phi + \eta^{\mathbf{ab}} \phi$$

Conclusions

1) Gauge invariant approach to
currents and shadows
give possibility to choice

**various gauges which might
be helpful in applications**

Standard currents are obtained
via Stueckelberg gauge fixing.

**But others gauges
might also be interesting**

2)

modified de Donder gauge leads to decoupled equations

and might be helpful for study

AdS/CFT

AdS/QCD

quantization of higher-spin AdS fields