Numerical Evaluation of Gauge Invariants for *a*-gauge Solutions in Open String Field Theory

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Collaboration with Tomohiko Takahashi (Nara Women's Univ.)

arXiv:0902.0445[hep-th], to appear in PTP121(2009)

cf. Kawano,I.K.,Takahashi, NPB803 (2008)135,arXiv:0804.1541

Non-perturbative vacuum in bosonic open string field theory

- Schnabl's solution $\Psi_{\rm Sch}$

Gauge invariants

(1) Action: D-brane tension

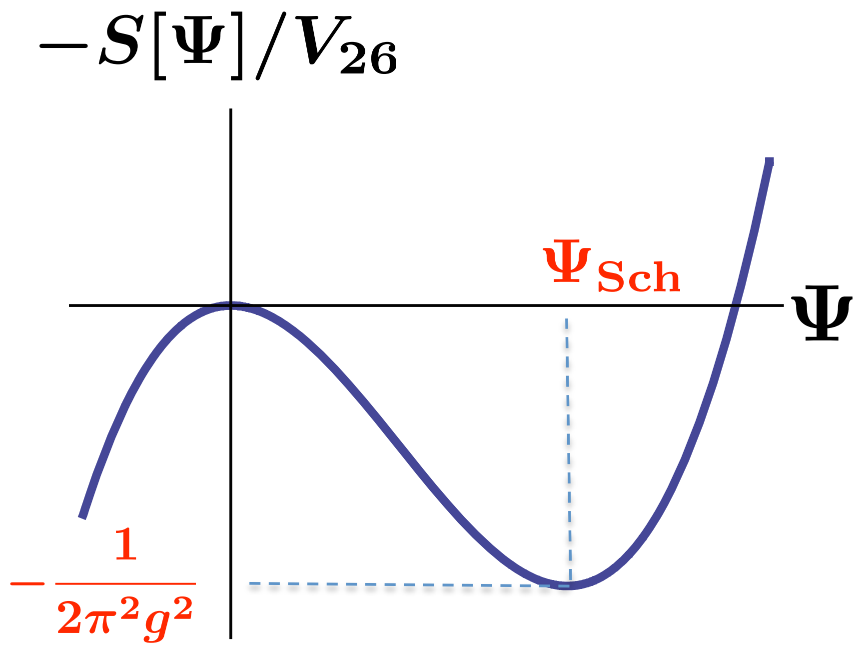
$$S[\Psi_{\rm Sch}]/V_{26} = \frac{1}{2\pi^2 g^2}$$

[Schnabl(2005),Okawa,Fuchs-Kroyter(2006)]

(2) Gauge invariant overlap:

$$\mathcal{O}_V(\Psi_{\rm Sch})/V_{26} = \frac{1}{2\pi}$$

[Ellwood, Kawano-Kishimoto-Takahashi(2008)]



Numerical solution by level truncation

- Numerical solution in the Siegel gauge : $b_0|\Psi_{\rm N}\rangle = 0$

  [...,Sen-Zwiebach(1999),...]

(1) $S[\Psi_{\rm N}]/S[\Psi_{\rm Sch}]$

(L,2L)-truncation	
(2,4)	0.9485534
(4,8)	0.9864034
(6,12)	0.9947727
(8,16)	0.9977795
(10,20)	0.9991161
(12,24)	0.9997907
(14,28)	1.0001580
(16,32)	1.0003678
(18,36)	1.00049

(L,3L)-truncation	
(2,6)	0.9593766
(4,12)	0.9878218
(6,18)	0.9951771
(8,24)	0.9979302
(10,30)	0.9991825
(12,36)	0.9998223
(14,42)	1.0001737
(16,48)	1.0003754
(18,54)	1.0004937

[Gaiotto-Rastelli(2002)]

(2) $\mathcal{O}_V(\Psi_{\rm N})/\mathcal{O}_V(\Psi_{\rm Sch})$

(L,2L)-truncation	
(2,4)	0.8783238
(4,8)	0.9294792
(6,12)	0.9501746
(8,16)	0.9606165
(10,20)	0.9677900
(12,24)	0.9723211
(14,28)	0.9760046
(16,32)	0.9785442

(L,3L)-truncation	
(2,6)	0.8898618
(4,12)	0.9319524
(6,18)	0.9510789
(8,24)	0.9611748
(10,30)	0.9681148
(12,36)	0.9725595
(14,42)	0.9761715
(16,48)	0.9786768

[Kawano-Kishimoto-Takahashi(2008)]
and the latest result

Evidence of gauge equivalence: $\Psi_{\rm N} \sim \Psi_{\rm Sch}$

Numerical solutions in a-gauges

- Asano-Kato's *a*-gauge
$$(b_0 M + a\, b_0 c_0 \tilde{Q})|\Psi_a\rangle = 0$$
$$Q = \tilde{Q} + c_0 L_0 + b_0 M$$

$a = 0$ ⇒ Siegel gauge: $b_0|\Psi_0\rangle = 0$

$a = \infty$ ⇒ Landau gauge: $b_0 c_0 \tilde{Q}|\Psi_\infty\rangle = 0$

(1) For *a*-gauge solution, (6,18)-trucation $S[\Psi_a]/S[\Psi_{\rm Sch}]$

$a = \infty$	0.9609438
$a = 4.0$	0.9244886
$a = 0.5$	1.0045858
$a = -2.0$	0.9798943
$\vdots$	$\vdots$

[Asano-Kato(2006)]

(and higher level?)

(2) $\mathcal{O}_V(\Psi_a)/\mathcal{O}_V(\Psi_{\rm Sch})$ (?)

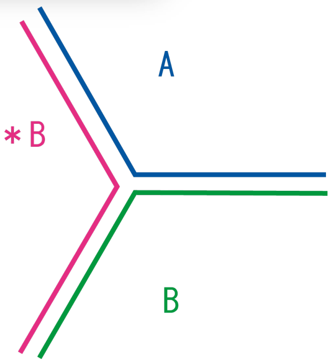
⇒ our computation

Contents

Bosonic cubic open string field theory

Action:
$$S[\Psi] = -\frac{1}{g^2}\left(\frac{1}{2}\langle\Psi, Q\Psi\rangle + \frac{1}{3}\langle\Psi, \Psi * \Psi\rangle\right)$$

$$Q = \oint \frac{dz}{2\pi i}\left(cT^{\rm m} + bc\partial c + \frac{3}{2}\partial^2 c\right)$$



Equation of motion:
$$Q\Psi + \Psi * \Psi = 0$$

Gauge transformation:
$$\delta_\Lambda \Psi = Q\Lambda + \Psi * \Lambda - \Lambda * \Psi$$

$$\rightarrow \quad \delta_\Lambda S[\Psi] = 0$$

Restrict string field to twist even sector in the universal space:
$$\Psi = (t_1 + t_2 b_{-1}c_{-1} + t_3 L^{(\rm m)}_{-2} + \cdots)c_1|0\rangle + (u_1 b_{-2} + \cdots)c_0 c_1|0\rangle$$

Gauge invariant overlap

Gauge invariant for on−shell closed string state

$$\mathcal{O}_V(\Psi) = \langle \mathcal{I} | V(i) | \Psi \rangle = \langle \hat{\gamma}(1_c, 2) | \Phi_V \rangle_{1_c} | \Psi \rangle_2$$

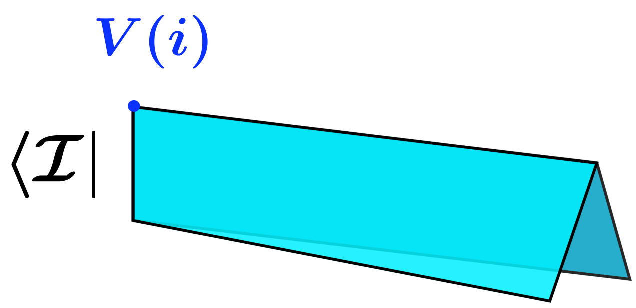
$$|\Phi_V\rangle = c_1 \bar{c}_1 |V_m\rangle$$

V_m :matter primary with (1,1)-dim.

$$\mathcal{O}_V(Q\Lambda) = 0$$

$$\mathcal{O}_V(\Psi * \Lambda) = \mathcal{O}_V(\Lambda * \Psi)$$

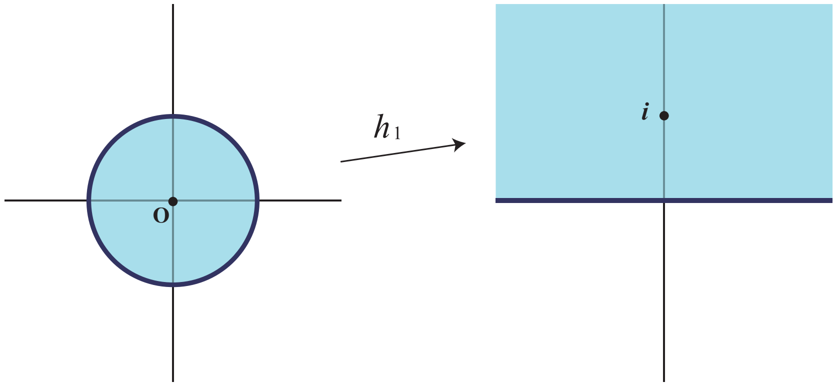
$$\Rightarrow \quad \delta_\Lambda \mathcal{O}_V(\Psi) = 0$$



In particular, it vanishes for pure gauge solutions: $\mathcal{O}_V(e^{-\Lambda} Q e^{\Lambda}) = 0$

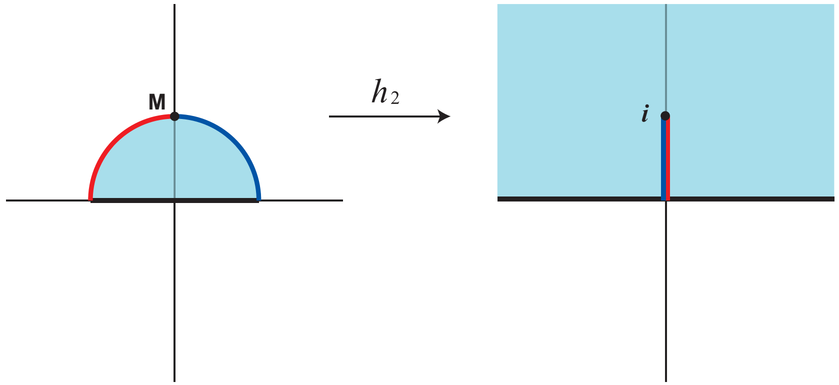
Shapiro-Thorn's vertex

$$\langle \hat{\gamma}(1_c, 2)|\phi_c\rangle_{1_c}|\psi\rangle_2 = \langle h_1[\phi_c(0)]h_2[\psi(0)]\rangle_{\mathrm{UHP}}$$



$$h_1(w) = -i\frac{w-1}{w+1}$$

closed string



$$h_2(w) = I \circ h_{\mathcal{I}}(w) = \frac{1}{2}\left(w - \frac{1}{w}\right)$$

open string

identity state: $\langle \mathcal{I}|\phi\rangle = \langle h_{\mathcal{I}}[\phi(0)]\rangle_{\mathrm{UHP}}$

Gauge invariant overlap for Schnabl's analytic solution

- Schnabl's solution for tachyon condensation

$$\Psi_{\rm Sch} = \frac{\partial_r}{e^{\partial_r}-1}\psi_r|_{r=0} = \sum_{n=0}^{\infty}\frac{B_n}{n!}\partial_r^n\psi_r|_{r=0}$$

$$= \lim_{N\to+\infty}\left(\psi_{N+1} - \sum_{n=0}^{N}\partial_r\psi_r|_{r=n}\right)$$

$$\psi_r \equiv \frac{2}{\pi}U^\dagger_{r+2}U_{r+2}\left[-\frac{1}{\pi}(\mathcal{B}_0+\mathcal{B}_0^\dagger)\tilde{c}(\frac{\pi r}{4})\tilde{c}(-\frac{\pi r}{4}) + \frac{1}{2}(\tilde{c}(-\frac{\pi r}{4})+\tilde{c}(\frac{\pi r}{4}))\right]|0\rangle \qquad U_r \equiv (2/r)^{\mathcal{L}_0}$$

$\mathcal{O}_V(\psi_r)$:independent of r

[Ellwood, Kawano-Kishimoto-Takahashi (2008)]

$$\mathcal{O}_V(\Psi_{\rm Sch}) = \mathcal{O}_V(\psi_0) = \lim_{N\to\infty}\mathcal{O}_V(\psi_{N+1})$$

Analytic computation of gauge inv. overlap for Schnabl's solution (1)

- Note: $\psi_r = \frac{2}{\pi} c_1|0\rangle + O(\mathcal{L}_0 - \mathcal{L}_0^\dagger, \mathcal{B}_0 - \mathcal{B}_0^\dagger, c_n + (-1)^n c_{-n})$

$\uparrow$ ψ_0

does not contribute to the gauge invariant overlap.

$$\langle\hat{\gamma}(1_c,2)|\left((L_n^{(2)} - (-1)^n L_{-n}^{(2)} - (-1)^{\frac{n}{2}}\frac{n}{4}c\,\delta_{n:\mathrm{even}}\right)$$

$$= \langle\hat{\gamma}(1_c,2)|(-2i^n)\sum_{m\geq 0}(-1)^m(\eta^n_{2m+1} - \eta^n_{2m-1})(L_m^{(1)} + (-1)^n \bar{L}_m^{(1)})$$

$$\langle\hat{\gamma}(1_c,2)|(b_n^{(2)} - (-1)^n b_{-n}^{(2)})$$

$$= \langle\hat{\gamma}(1_c,2)|(-2i^n)\sum_{m\geq 0}(-1)^m(\eta^n_{2m+1} - \eta^n_{2m-1})(b_m^{(1)} + (-1)^n \bar{b}_m^{(1)})$$

$$\langle\hat{\gamma}(1_c,2)|(c_m^{(2)} + (-1)^m c_{-m}^{(2)})$$

$$= \langle\hat{\gamma}(1_c,2)|\frac{-i^m}{4}\sum_{n\geq 1}(-1)^n(\eta^{2n}_{m+1} - \eta^{2n}_{m-1} + \delta_{m,1})(c_n^{(1)} + (-1)^m \bar{c}_n^{(1)})$$

$$\left(\frac{1+x}{1-x}\right)^k = \sum_{n=0}^{\infty}\eta_n^k x^n$$

Analytic computation of gauge inv. overlap for Schnabl's solution (2)

- Relation to the boundary state

$$\langle\hat{\gamma}(1_c,2)|\psi_0\rangle_2\mathcal{P}_{1_c}=\frac{1}{2\pi}\langle B|c_0^-$$

[Kawano-I.K.-Takahashi(2008)]

generalization

[Kiermaier-Okawa-Zwiebach(2008)]

$$\begin{aligned}|B_*(\Psi_{\mathrm{Sch}})\rangle &\equiv e^{\frac{\pi^2}{s}(L_0+\bar{L}_0)}\oint_s \mathrm{P}e^{-\int_0^s dt[\mathcal{L}_R(t)+\{\mathcal{B}_R(t),\Psi_{\mathrm{Sch}}\}]}\\ &= |B\rangle+\sum_{k=1}^{\infty}|B_*^{(k)}(\Psi_{\mathrm{Sch}})\rangle\\ &= 0\end{aligned}$$

Gauge invariant overlap for string fields in the universal space

- For string fields in the twist even universal space such as

$$\begin{aligned}\Psi_{\rm univ} &= (t_1 + t_2 b_{-1}c_{-1} + t_3 L^{(\rm m)}_{-2} + t_4 b_{-3}c_{-1} + t_5 b_{-2}c_{-2} + t_6 b_{-1}c_{-3} \\ &\quad + t_7 L^{(\rm m)}_{-2} b_{-1}c_{-1} + t_8 L^{(\rm m)}_{-4} + t_9 (L^{(\rm m)}_{-2})^2 + \cdots)c_1|0\rangle \\ &\quad + (u_1 b_{-2} + u_2 b_{-4} + u_3 b_{-2}b_{-1}c_{-1} + u_4 L^{(\rm m)}_{-2} b_{-2} + u_5 L^{(\rm m)}_{-3} b_{-1} + \cdots)c_0 c_1|0\rangle\end{aligned}$$

$$\mathcal{O}_V(\Psi_{\rm univ}) = \frac{1}{4}t_1 - \frac{1}{4}t_2 - \frac{3}{4}t_3 + \frac{1}{4}t_5 + \frac{3}{4}t_7 + \frac{3}{2}t_8 + \frac{11}{2}t_9 + \cdots$$

- Here, we take a normalization such as

$$|V_{\rm m}\rangle = \frac{-1}{26}\eta_{\mu\nu}\alpha^{\mu}_{-1}\bar{\alpha}^{\nu}_{-1}|0\rangle$$

Asano-Kato's *a*-gauge

In the worldsheet ghost number 1 sector,

$$(b_0 M + a b_0 c_0 \tilde{Q})\Phi_1 = 0$$

$$M = -2\sum_{n=1}^{\infty} n c_{-n} c_n$$

$$\tilde{Q} = \sum_{n\neq 0} c_{-n} L_n^{(\mathrm{m})} - \frac{1}{2}\sum_{n,m,m+n\neq 0} (m-n) c_{-m} c_{-n} b_{m+n}$$

Note: $a = 1$ ⇒ $b_0 c_0 Q \Phi_1 = 0$

Under the gauge transformation in the free level $\Phi_1 \mapsto \Phi_1 + Q\Lambda_0$
this condition cannot fix the gauge.

⇒ $a \neq 1$ *perturbatively*

On the *a*-gauge

- The *a*-gauge condition conserves the level.

  ⟹ suitable to the level truncation

- The *a*-gauge condition is compatible with the twist even sector in the universal space.

dimension of the truncated space in the *a*-gauge:

L	0	2	4	6	8	10	12	14	16	18
dim.	1	3	9	26	69	171	402	898	1925	3985

the same as that of the Siegel gauge

Asano-Kato's gauge fixed action

$$S_{\rm GF} = -\frac{1}{2}\sum_{n=-\infty}^{\infty}\langle \Phi_n, Q\Phi_{2-n}\rangle - \frac{g}{3}\sum_{l+n+m=3}\langle \Phi_l, \Phi_m * \Phi_n\rangle + \sum_{n=-\infty}^{\infty}\langle (\mathcal{O}_a\mathcal{B})_{3-n}, \Phi_n\rangle$$

$\Phi_n, \mathcal{B}_n$: worldsheet ghost number *n*

$$(\mathcal{O}_a\mathcal{B})_n = (b_0 M^{n-1} + a c_0 b_0 M^{n-2}\tilde{Q})\mathcal{B}_{3-n}$$

$$(\mathcal{O}_a\mathcal{B})_{3-n} = (b_0 W_{n-2} + a c_0 b_0 W_{n-1}\tilde{Q})\mathcal{B}_n \qquad (n \geq 2)$$

$$W_n = \sum_{i=0}^{\infty}\frac{(-1)^i(n+i-1)!}{i!(n-1)!((n+i)!)^2}M^i(M^-)^{n+i} \qquad M^- = -\sum_{n=1}^{\infty}\frac{1}{2n}b_{-n}b_n$$

integrate out $\mathcal{B}_n$

$$b_0(M^{n-1} + a c_0\tilde{Q}M^{n-2})\Phi_{3-n} = 0$$

$$b_0(W_{n-2} + a c_0\tilde{Q}W_{n-1})\Phi_n = 0 \qquad (n \geq 2)$$

gauge fixing condition

Massless part

Let us consider "level 1" part of the string fields:

$$\begin{aligned}\Phi &= \gamma(x)|0\rangle + (A_\mu(x)\alpha_{-1}^\mu c_1 + \beta(x)c_0)|0\rangle \\ &\quad +(\bar{\gamma}(x)c_{-1}c_1 + u_\mu(x)\alpha_{-1}^\mu c_0 c_1)|0\rangle + v(x)c_{-1}c_0c_1|0\rangle\end{aligned}$$

$$\mathcal{B} = \beta_\chi(x)c_0|0\rangle + \beta_\mu(x)\alpha_{-1}^\mu c_0c_1|0\rangle + \beta_v(x)c_{-1}c_0c_1|0\rangle$$

$$\begin{aligned}S_{\rm GF}|_{\rm quad.} &= \int d^{26}x\Big(-\frac{\alpha'}{4}(\partial_\mu A_\nu - \partial_\nu A_\mu)(\partial^\mu A^\nu - \partial^\nu A^\mu) - \frac{1}{2}(-\sqrt{2}i\beta + \sqrt{\alpha'}\partial_\mu A^\mu)^2 \\ &\quad - \alpha'\bar{\gamma}\partial_\mu\partial^\mu\gamma - i\sqrt{2\alpha'}u_\mu\partial^\mu\gamma \\ &\quad + \frac{1}{2}\beta_v v + \beta_\mu(u^\mu + a\sqrt{\alpha'/2}i\partial^\mu\bar{\gamma}) - \sqrt{2}i\beta_\chi(-\sqrt{2}i\beta + a\sqrt{\alpha'}\partial_\mu A^\mu)\Big)\end{aligned}$$

field redefinition

$$S_{\rm GF}|_{\rm quad.} = \int d^{26}x\Big(-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + B\partial_\mu A^\mu + \frac{\alpha}{2}B^2 + i\bar{c}\partial_\mu\partial^\mu c - \frac{1}{2}\tilde{\chi}^2 + \frac{1}{2}\tilde{\beta}_\mu\tilde{u}^\mu + \frac{1}{2}\beta_v v\Big)$$

$$\alpha = \frac{1}{(a-1)^2}$$

Construction of numerical solutions

$$\Psi_{(0)} = \frac{64}{81\sqrt{3}} c_1|0\rangle$$: nontrivial solution for (0,0)-truncation

$$(b_0 M + a\, b_0 c_0 \tilde{Q})\Psi_{(n+1)} = 0$$: *a*-gauge condition

$$\mathcal{P}(Q_{\Psi_{(n)}}\Psi_{(n+1)} - \Psi_{(n)} * \Psi_{(n)}) = 0$$ **: linear equations!**

$\mathcal{P} = c_0 b_0$: a projection to solve equations

$$Q_{\Psi_{(n)}}\Phi \equiv Q\Phi + \Psi_{(n)} * \Phi - (-1)^{|\Phi|}\Phi * \Psi_{(n)}$$

: "BRST operator" around $\Psi_{(n)}$

We can define $\Psi_{(n)} \mapsto \Psi_{(n+1)}$

$$\Psi_{(n+1)} \simeq (Q_{\Psi_{(n)}})^{-1}(\Psi_{(n)} * \Psi_{(n)})$$ [Gaiotto-Rastelli(2002)]

On the equation of motion

If the iteration converges for $n \longrightarrow \infty$

$$(b_0 M + a\, b_0 c_0 \tilde{Q})\Psi_{(\infty)} = 0$$: *a*-gauge condition

$$\mathcal{P}(Q\Psi_{(\infty)} + \Psi_{(\infty)} * \Psi_{(\infty)}) = 0$$: projected part of eq. of motion

We check the remaining part of the equation of motion
for the resulting configuration:

$$\underbrace{(1-\mathcal{P})}_{b_0 c_0}(Q\Psi_{(\infty)} + \Psi_{(\infty)} * \Psi_{(\infty)}) = 0 \quad (?)$$

"BRST invariance"

[Hata-Shinohara(2000)]

"Norm" of string fields

Level *L*-truncated string field in the universal space:

$$\Phi = \sum_{k+l\leq L}\sum_{m_k,n_l} t_{k,m_k;l,n_l}\,\varphi_{k,m_k}\otimes\psi_{l,n_l}$$

φ_{k,m_k} : a linear combination of

$$L^{(\mathrm{m})}_{-n_1}L^{(\mathrm{m})}_{-n_2}\cdots L^{(\mathrm{m})}_{-n_q}|0\rangle_{\mathrm{m}}\quad (n_1\geq n_2\geq\cdots\geq n_q\geq 2)$$

s.t.

$$\langle\varphi_{k,m_k},\varphi_{k',m'_{k'}}\rangle = (-1)^k\delta_{k,k'}\delta_{m_k,m'_{k'}},\quad L_0^{(\mathrm{m})}|\varphi_{k,m_k}\rangle = k|\varphi_{k,m_k}\rangle$$

$$|\psi_{k,m_k}\rangle = b_{-p_1}b_{-p_2}\cdots b_{-p_r}c_{-q_1}c_{-q_2}\cdots c_{-q_s}c_1|0\rangle_{\mathrm{gh}}$$

$$p_1>p_2>\cdots>p_r\geq 1,\quad q_1>q_2>\cdots>q_s\geq 0,\quad \sum_{t=1}^{r}p_t+\sum_{u=1}^{s}q_u=k$$

$$\Rightarrow\qquad \|\Phi\| = \left(\sum_{k,m_k,l,n_l}|t_{k,m_k;l,n_l}|^2\right)^{\frac{1}{2}}$$

Convergence of iterations

We continue the iterations until

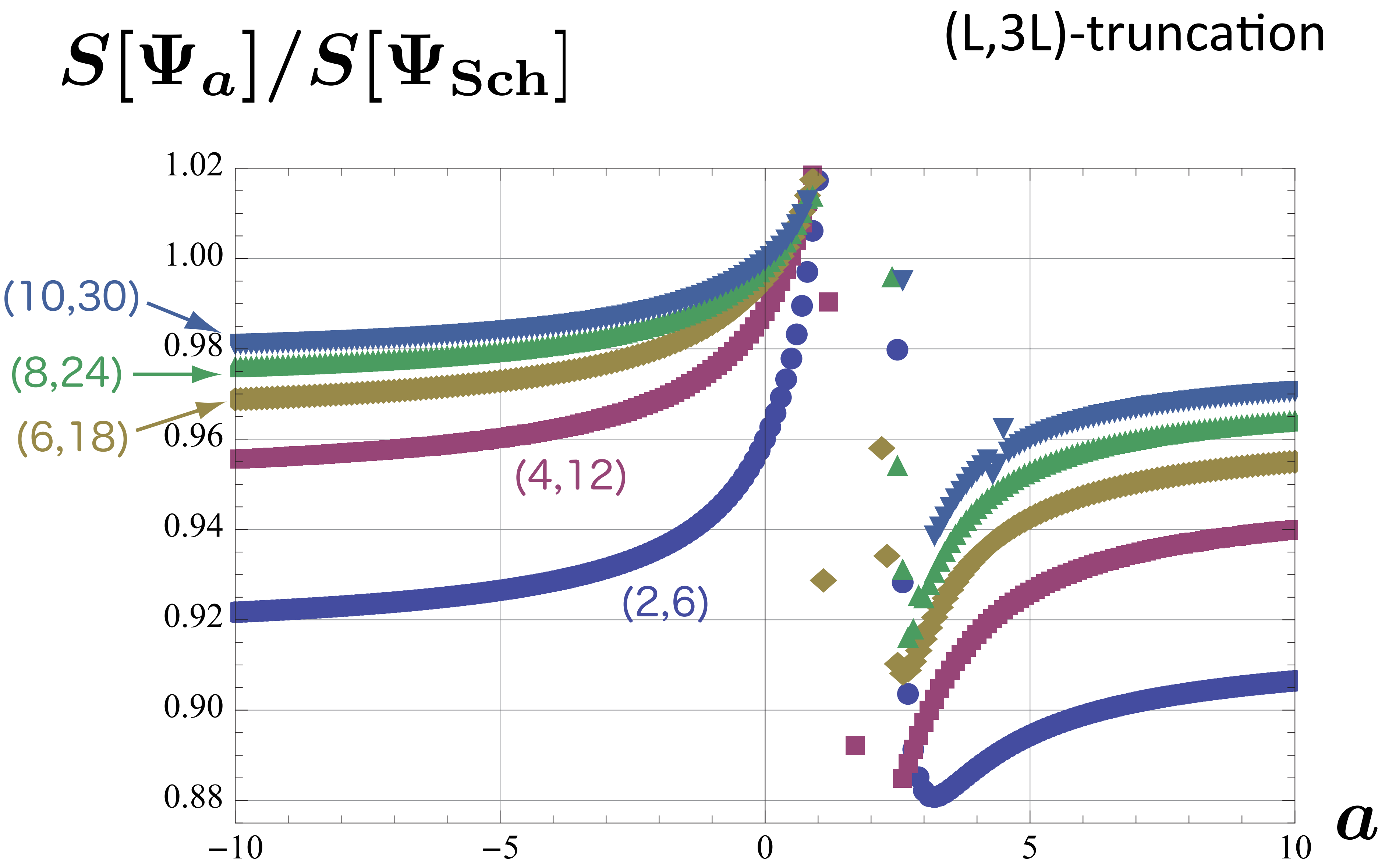
$$\frac{||\Psi_M - \Psi_{M-1}||}{||\Psi_M||} < 10^{-8}$$

For various *a*, $M < 10$

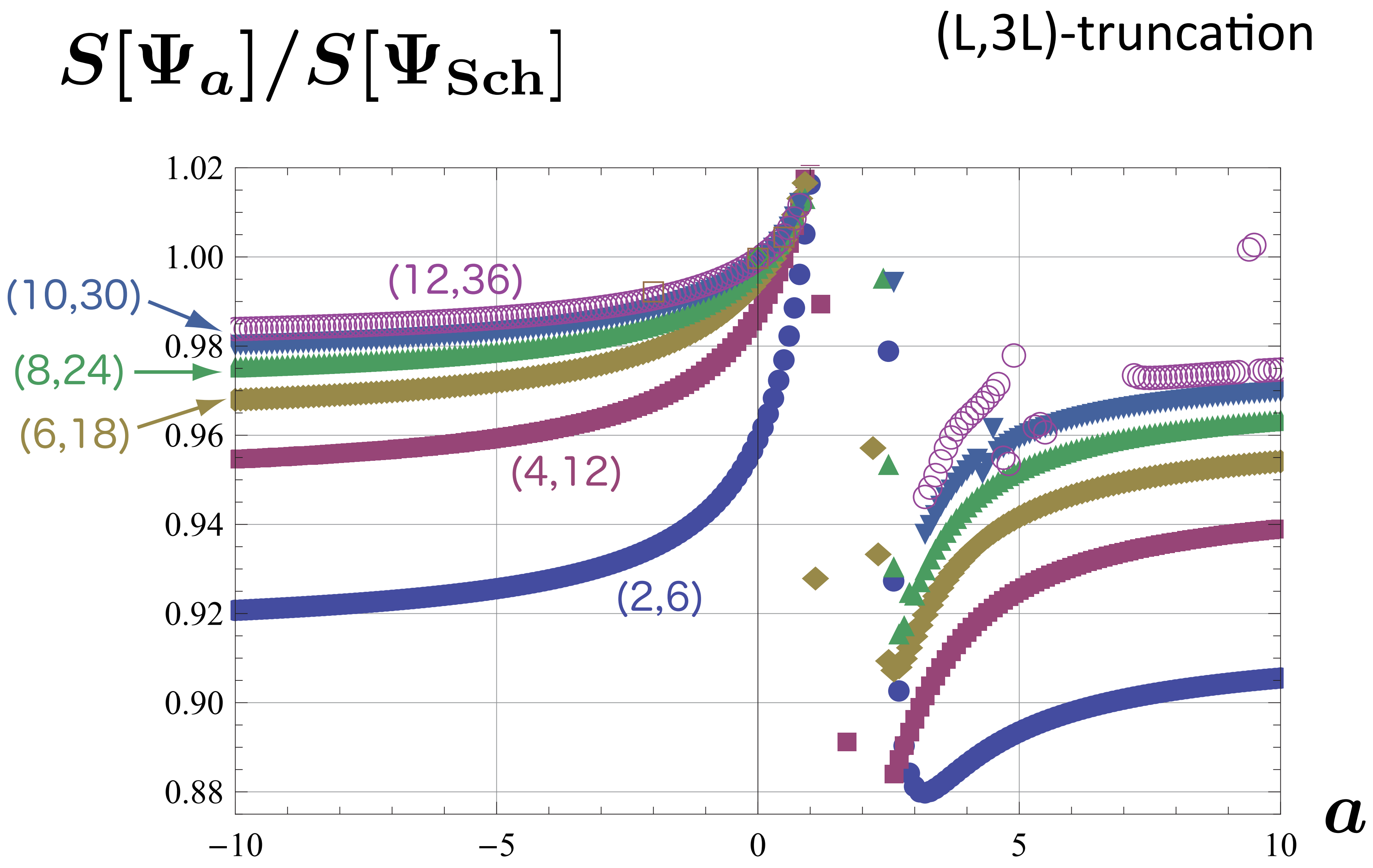
$$-\infty \leqq a \lesssim 0, \ 1 \ll a \leqq \infty$$

$$||\Psi_M|| \sim O(1)$$

$$\frac{||c_0 b_0 (Q\Psi_M + \Psi_M * \Psi_M)||}{||\Psi_M||} < 10^{-8}$$



$S[\Psi_a]/S[\Psi_{\mathbf{Sch}}]$
(L,3L)-truncation
(10,30)
(8,24)
(6,18)
(4,12)
(2,6)
1.02
1.00
0.98
0.96
0.94
0.92
0.90
0.88
−10
−5
0
5
10
a



$S[\Psi_a]/S[\Psi_{\mathbf{Sch}}]$
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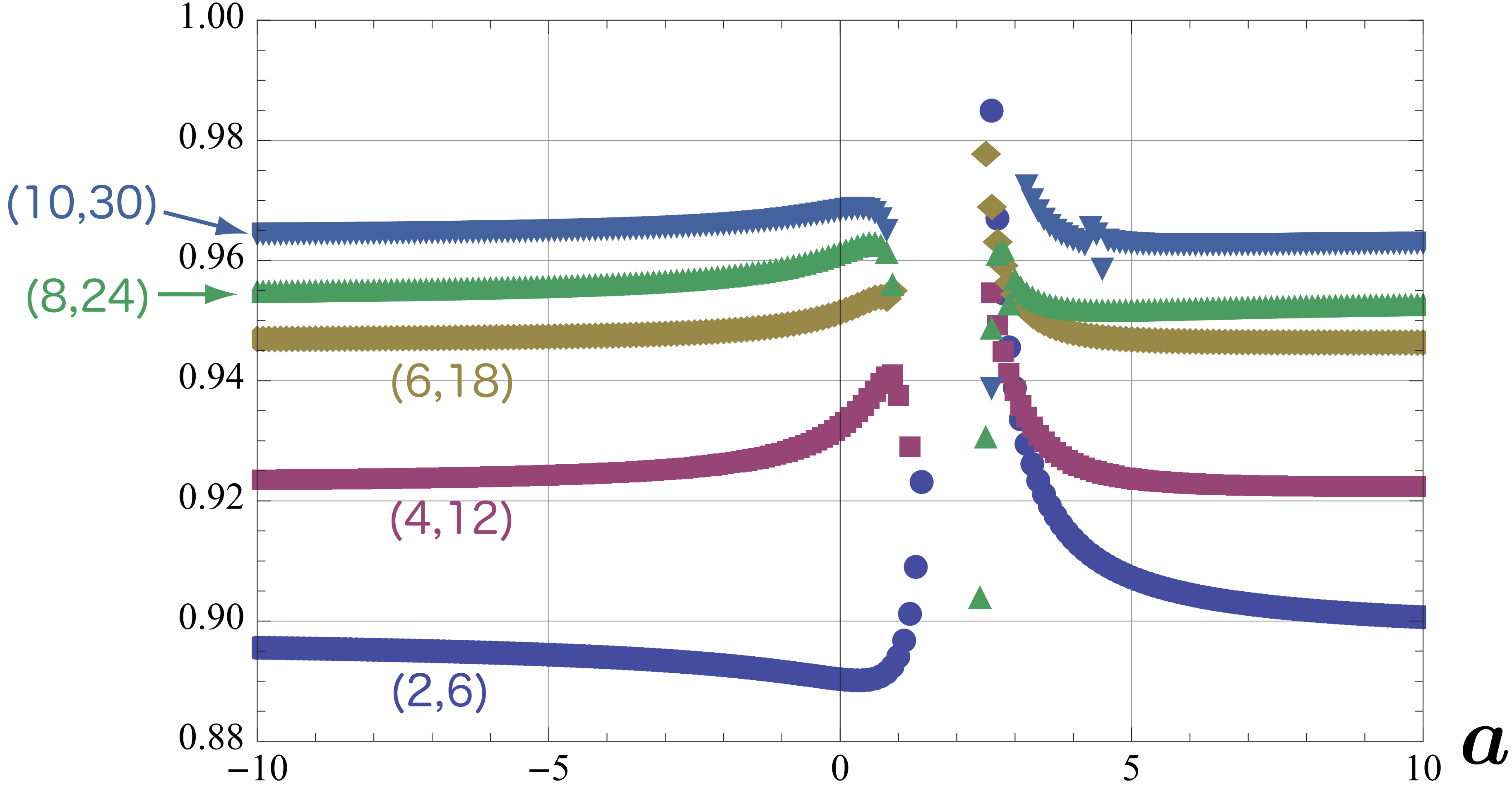
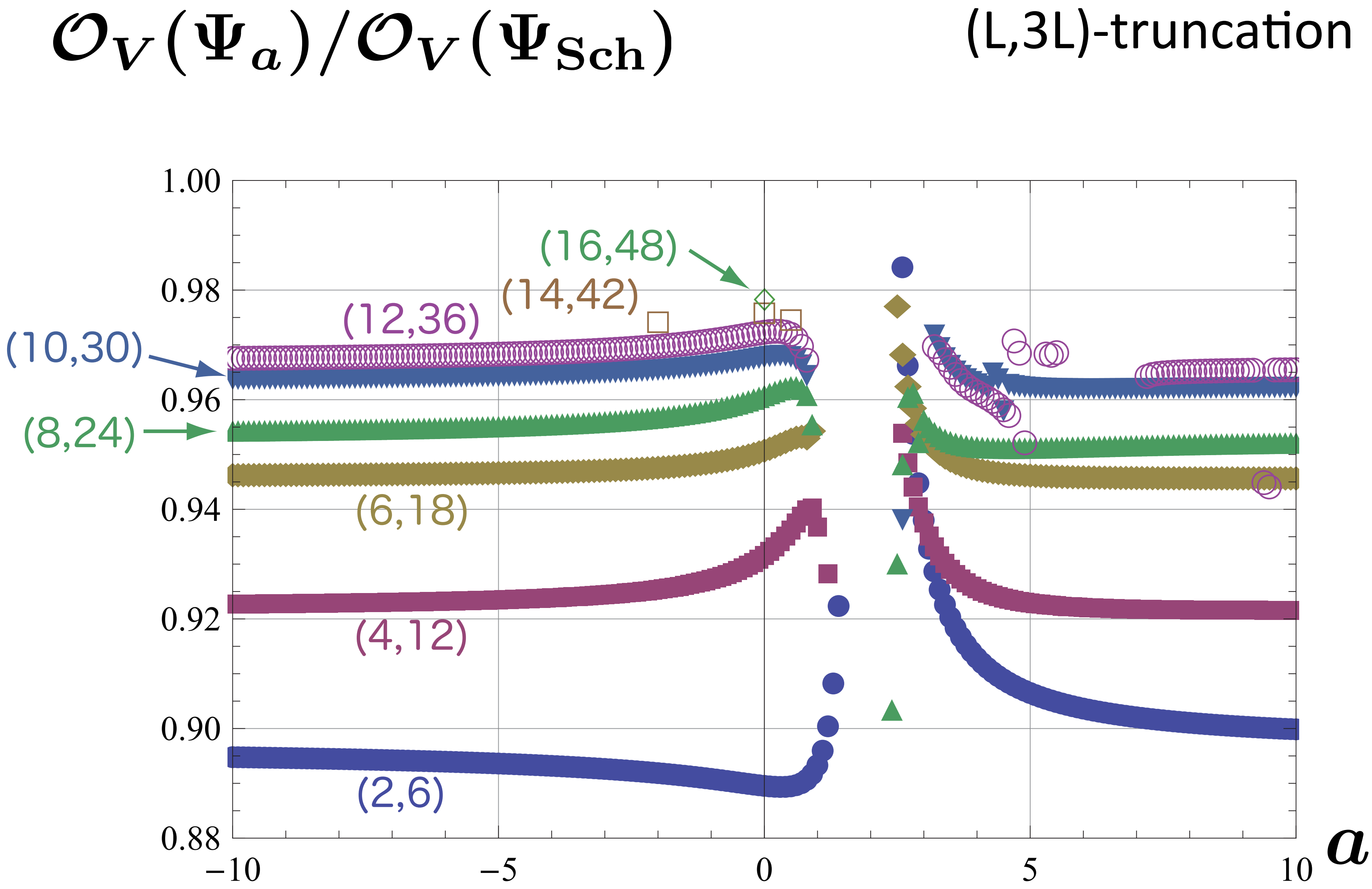
5
10
a

$\mathcal{O}_V(\Psi_a)/\mathcal{O}_V(\Psi_{\mathbf{Sch}})$

(L,3L)-truncation





$\mathcal{O}_V(\Psi_a)/\mathcal{O}_V(\Psi_{\mathbf{Sch}})$
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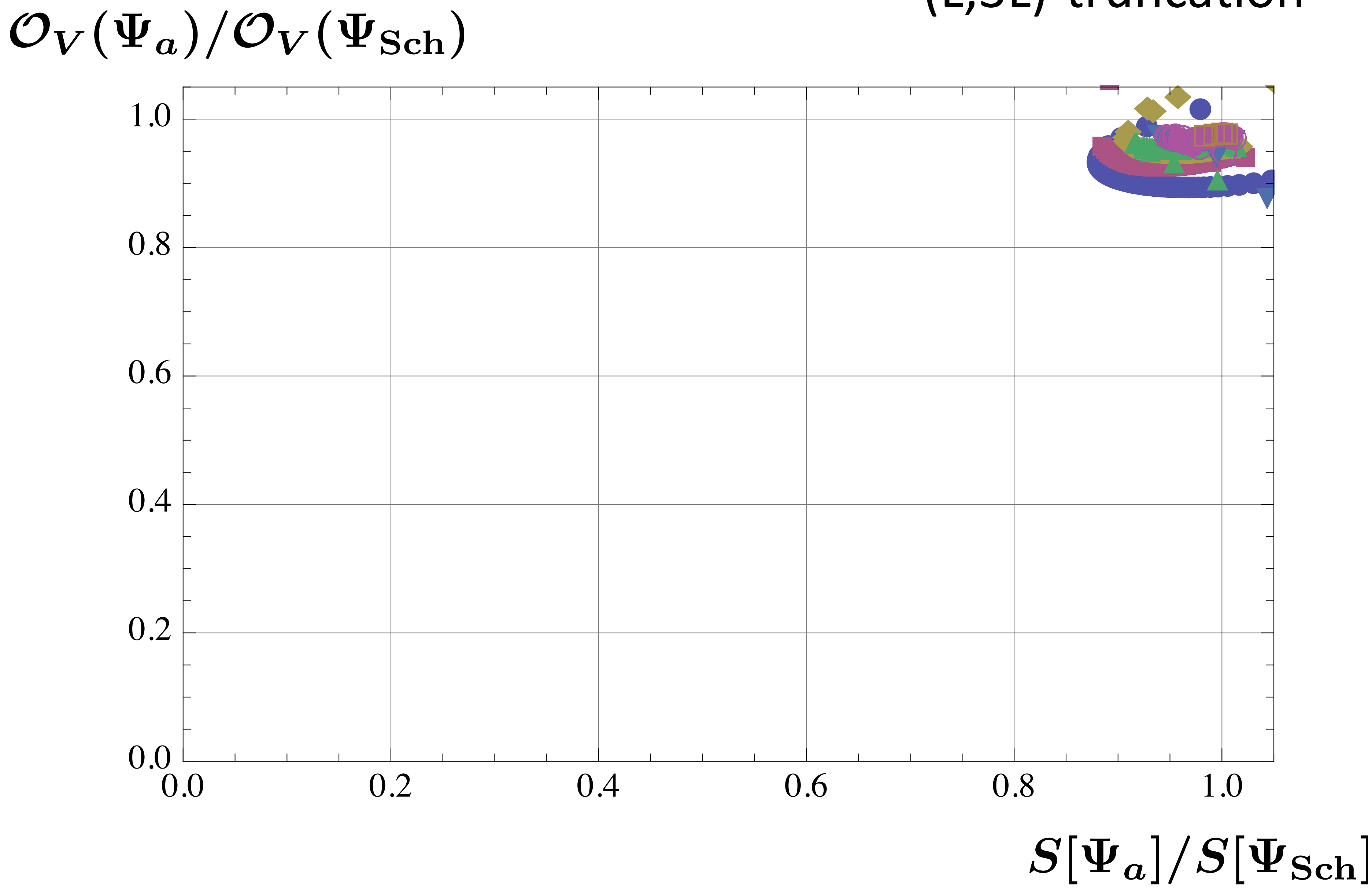
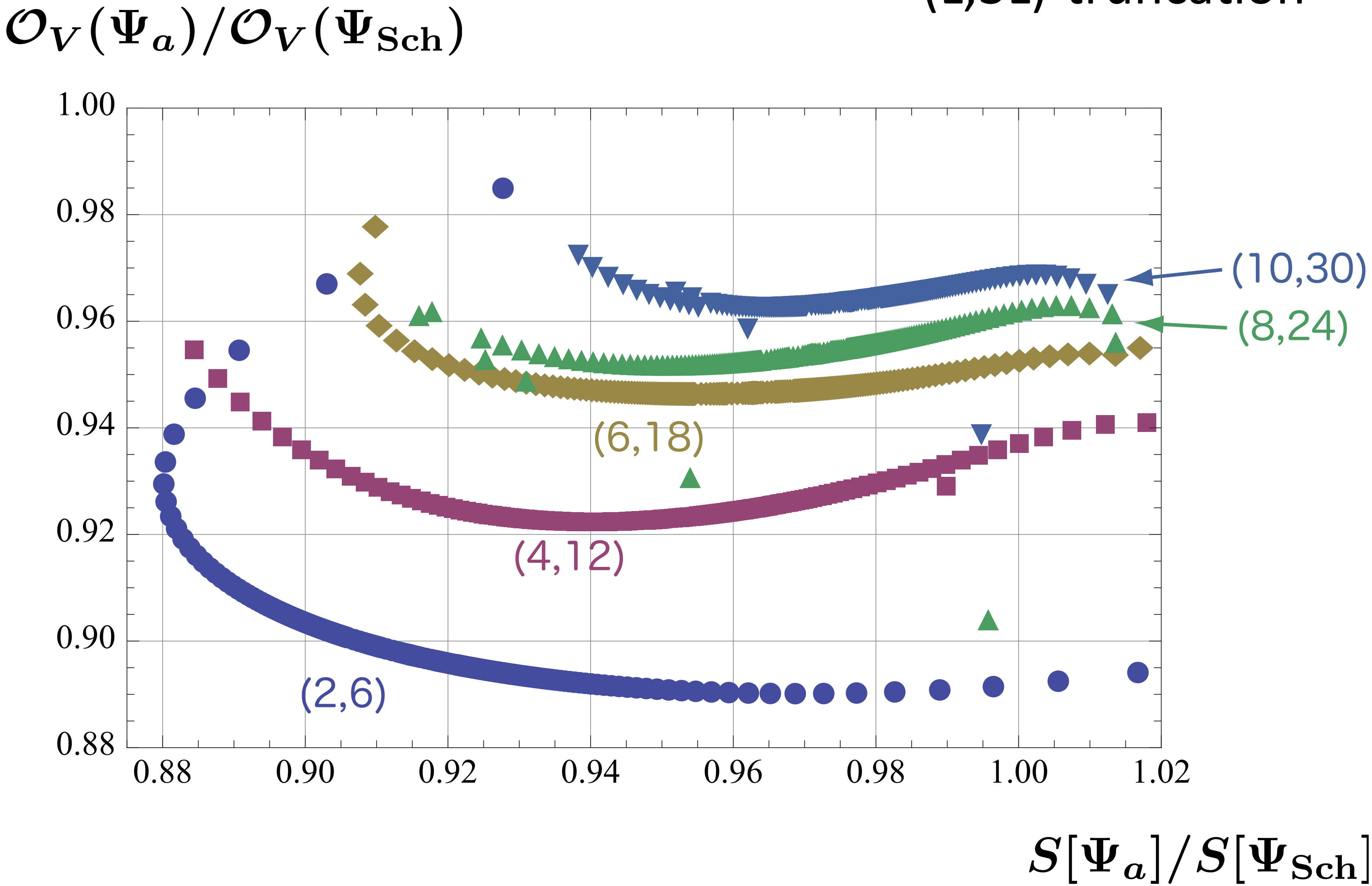
Gauge invariants for various *a*-gauge solutions

(L,3L)-truncation



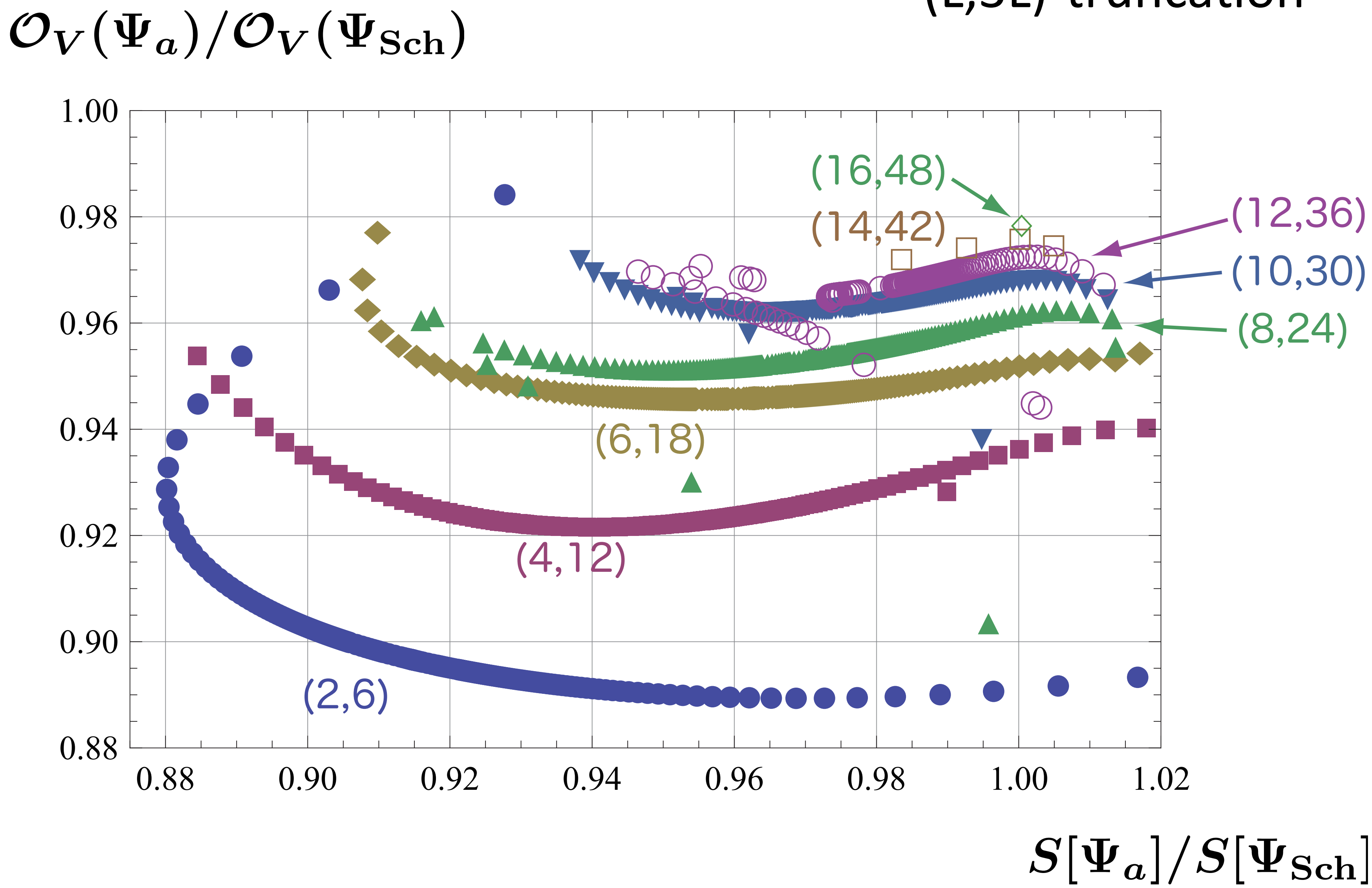
Gauge invariants for various a-gauge solutions

(L,3L)-truncation



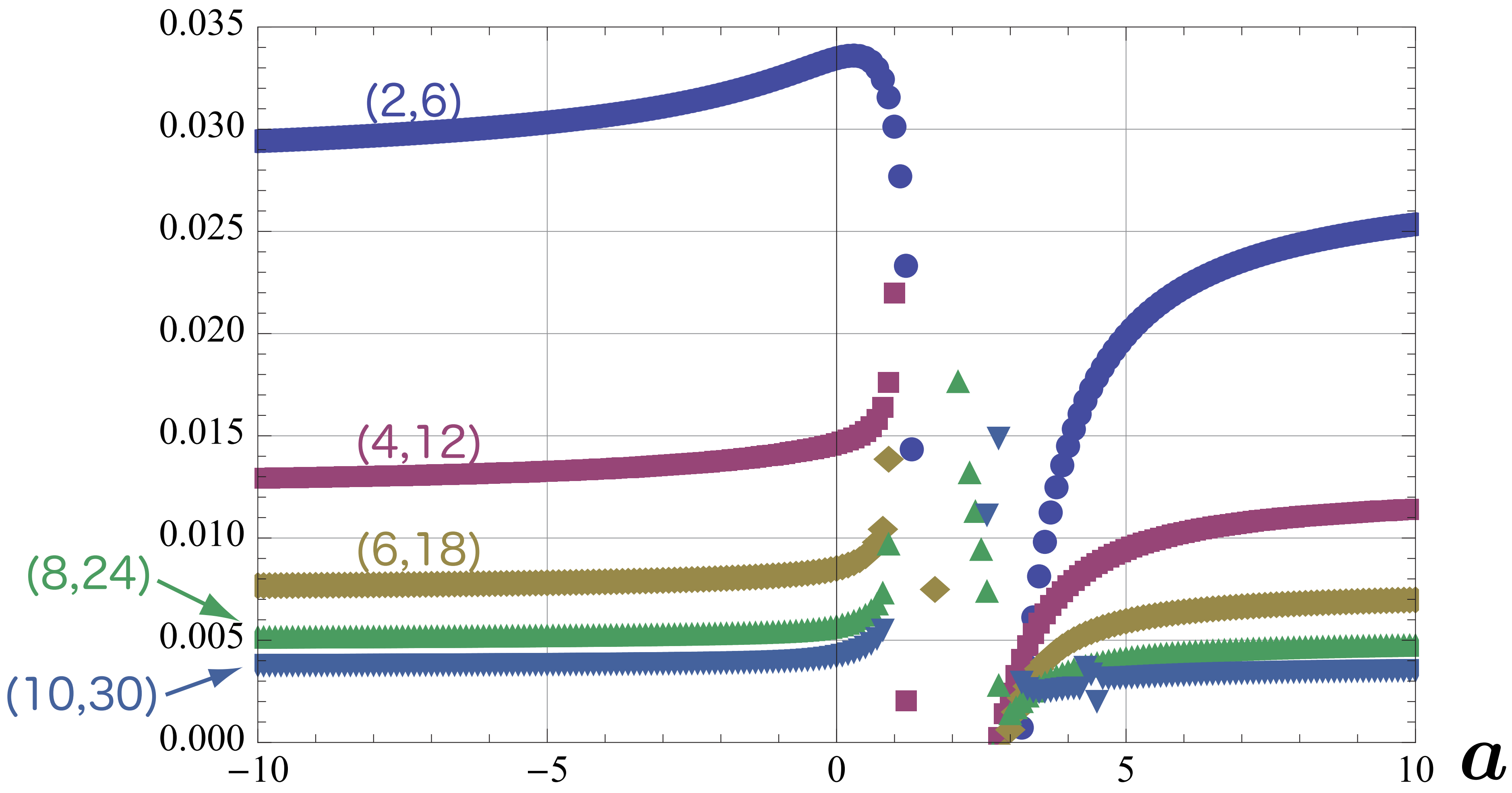
Gauge invariants for various a-gauge solutions

(L,3L)-truncation



Coefficient of $c_{-2}c_1|0\rangle \in (1-\mathcal{P})(Q\Psi_a+\Psi_a*\Psi_a)$

(L,3L)-truncation



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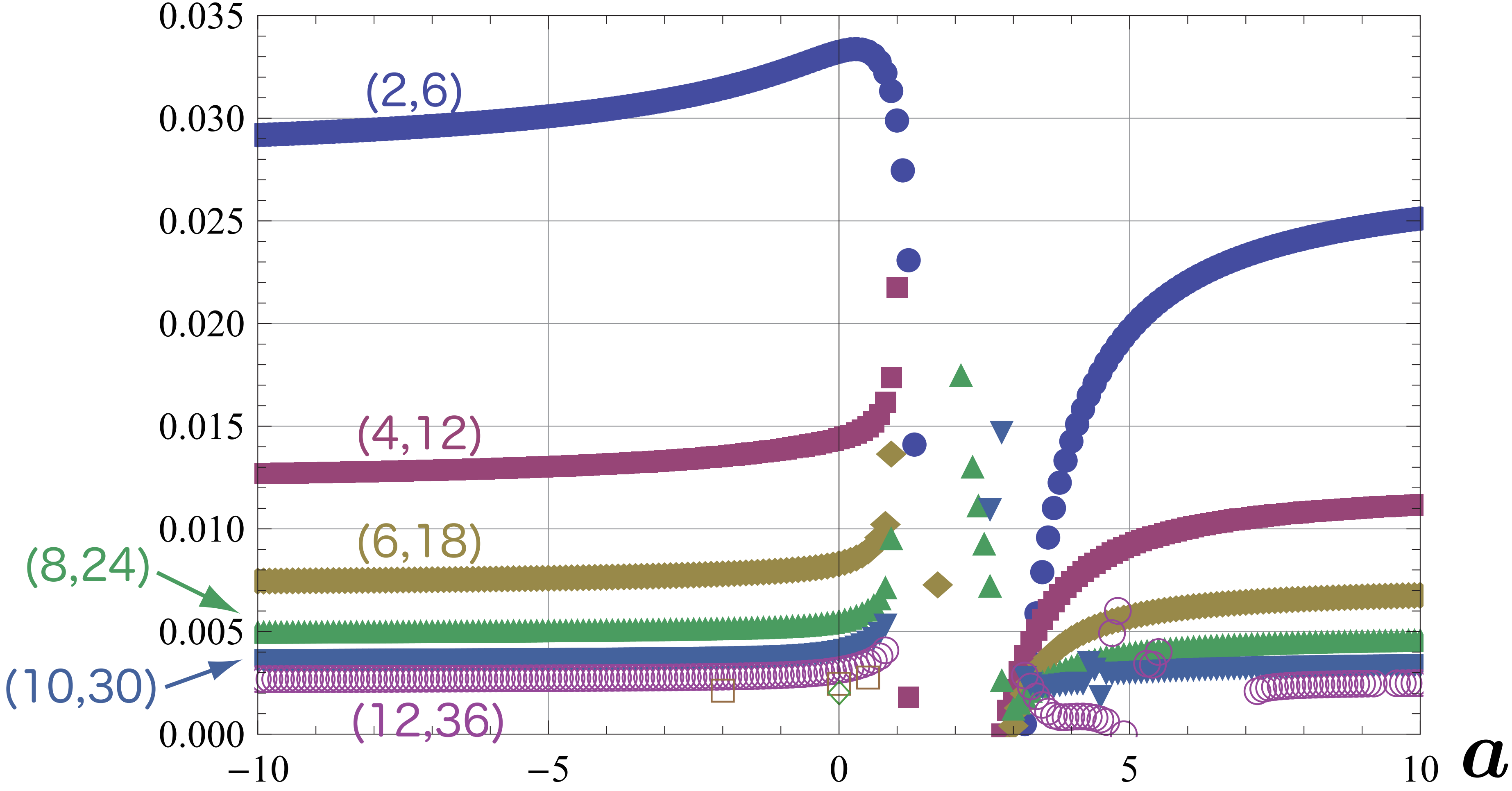
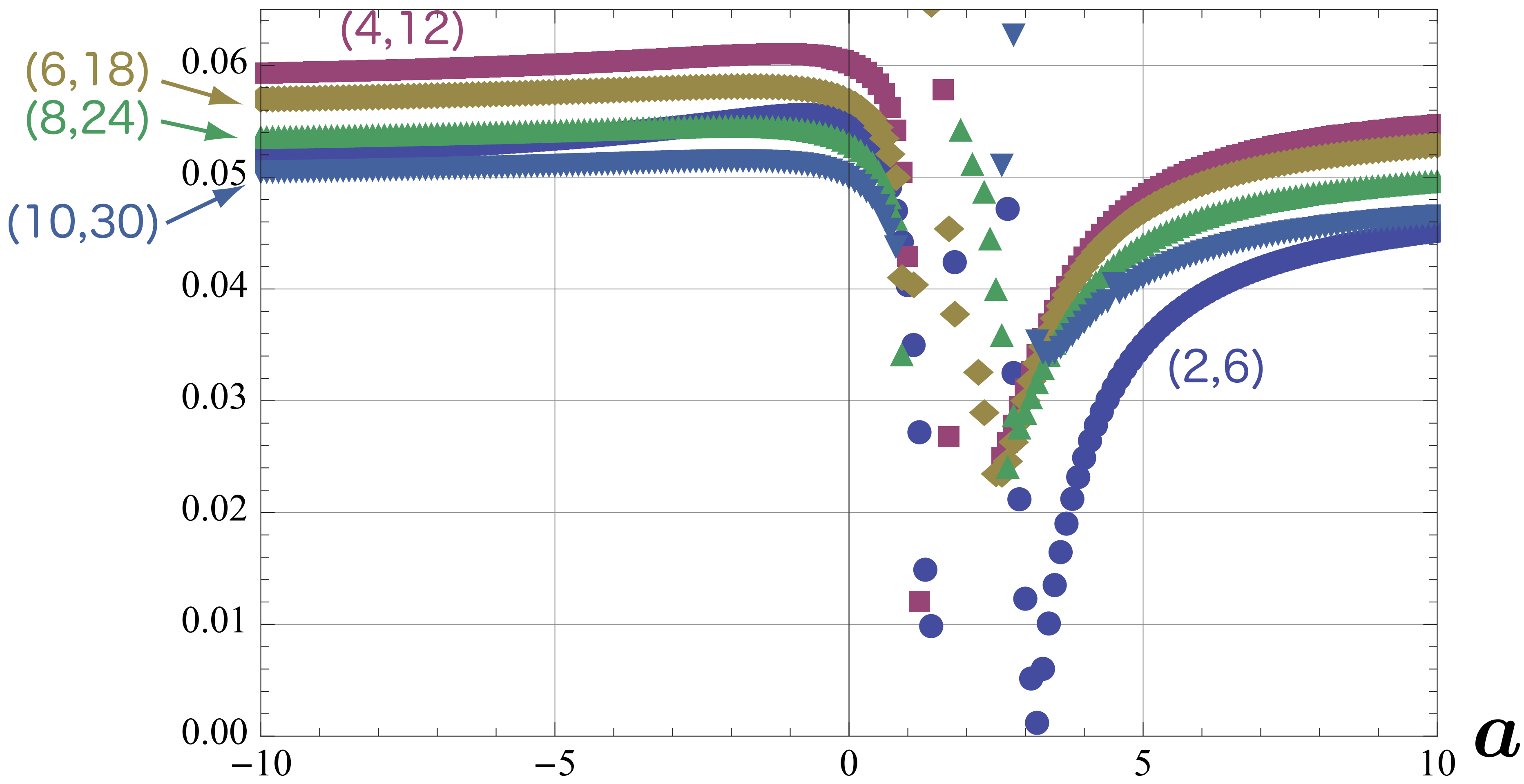


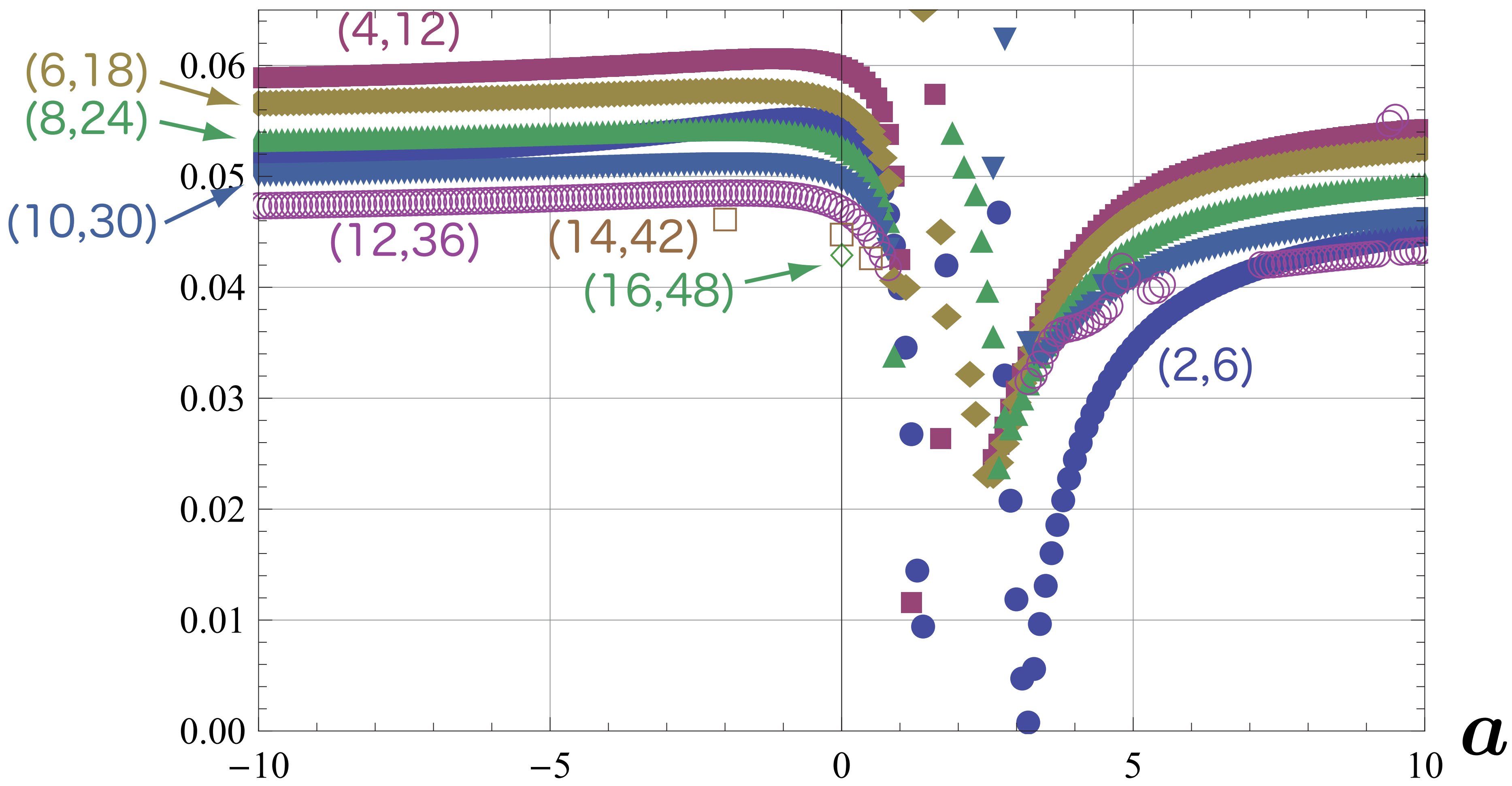
$$\frac{\|(1-\mathcal{P})(Q\Psi_a + \Psi_a * \Psi_a)\|}{\|\Psi_a\|}$$

(L,3L)-truncation



$$\frac{\|(1-\mathcal{P})(Q\Psi_a+\Psi_a*\Psi_a)\|}{\|\Psi_a\|}$$

(L,3L)-truncation



Summary

- We have evaluated gauge invariants (action and gauge invariant overlap) for numerical solutions in *a*-gauges by level truncation ((L,2L) and (L,3L)-method).
- We have checked the consistency of the equation of motion.
- Our numerical results suggest: $-\infty \leqq a \lesssim 0,\ 1 \ll a \leqq \infty$

$$L \to +\infty \qquad \begin{array}{c} S[\Psi_{a,L}]|_L \longrightarrow S[\Psi_{\mathrm{Sch}}] \\ \mathcal{O}_V(\Psi_{a,L}) \longrightarrow \mathcal{O}_V(\Psi_{\mathrm{Sch}}) \end{array}$$

- These are consistent with the gauge equivalence:

$$\Psi_a \sim \Psi_{\mathrm{Sch}}$$

Discussion

- The approaching speed of the overlap to the expected value is slower than that of the action.
- Due to the subtlety of the midpoint(?)

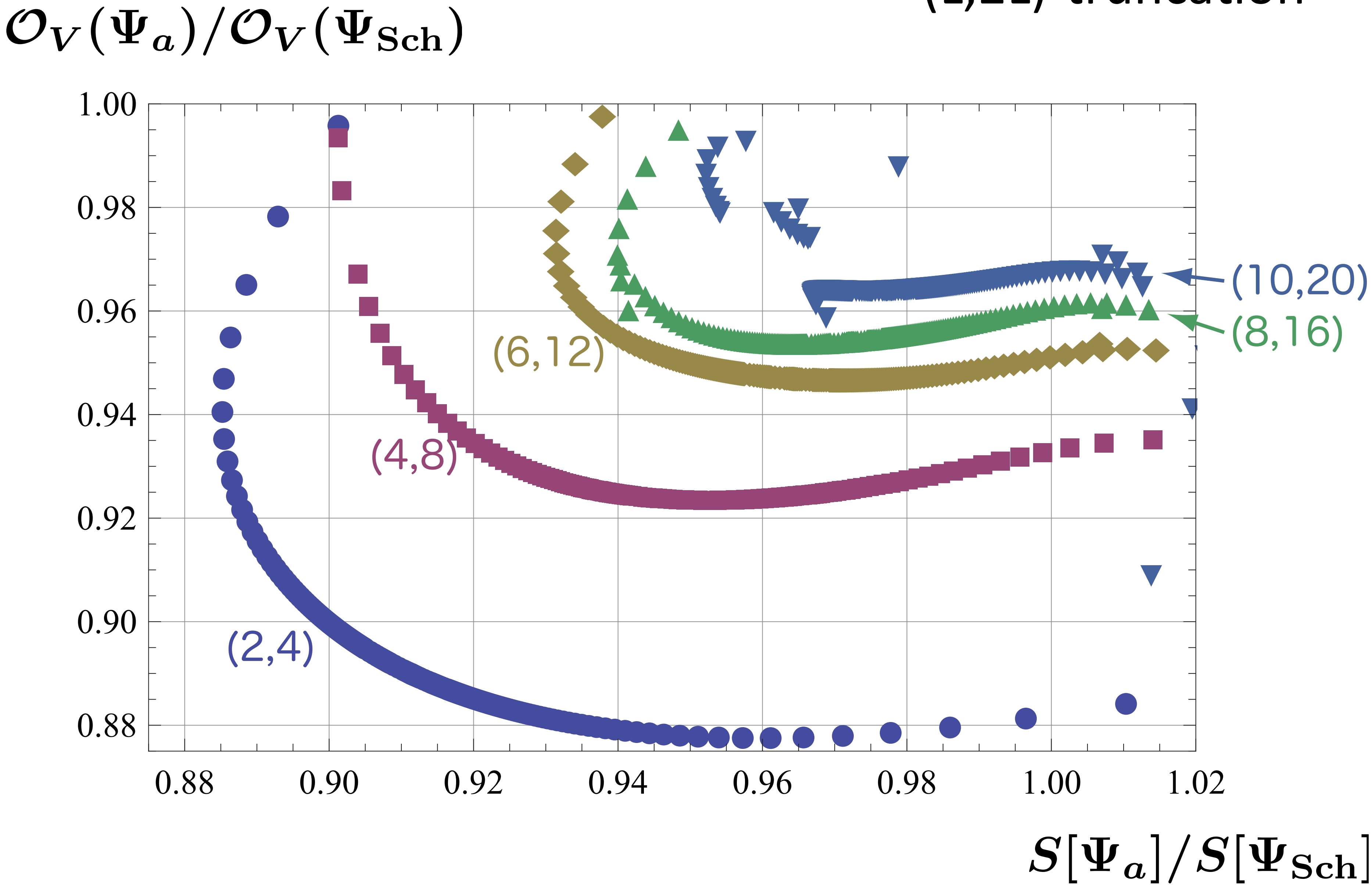
(Suppose that the gauge invariant overlap is always well-defined.)

- If there is a small discrepancy between the gauge invariant overlap for the *a*-gauge solutions and that for the Schnabl solution, they are not gauge equivalent.

If so, they might describe different vacua. (!?)

Gauge invariants for various *a*-gauge solutions

(L,2L)-truncation



Gauge invariants for various *a*-gauge solutions

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