Pure Gauge Configurations and Tachyon Solutions to String Field Theories Equations of Motion

Grigoryev Dmitry

Steklov Mathematical Institute RAS

April 13, 2009

Set Up

- $\langle\langle ... \rangle\rangle = \langle Y_{-2} ... \rangle$
- Action for fermionic string

$$S(\Phi_+, \Phi_-) = \frac{1}{2}\langle\langle \Phi_+, Q\Phi_+ \rangle\rangle + \frac{1}{2}\langle\langle \Phi_-, Q\Phi_- \rangle\rangle + \frac{1}{3}\langle\langle \Phi_+, \Phi_+ * \Phi_+ \rangle\rangle - \langle\langle \Phi_+, \Phi_- * \Phi_- \rangle\rangle \quad (1.1)$$

- Equations of motion

$$\begin{cases} Q\Phi_+ + \Phi_+ \star \Phi_+ - \Phi_- \star \Phi_- = 0, \\ Q\Phi_- + \Phi_+ \star \Phi_- - \Phi_- \star \Phi_+ = 0. \end{cases} \quad (1.2)$$

Relations needed for calculations

$$
\begin{aligned}
\{B, c\} &= 1, \quad [K, B] = 0, \quad B^2 = c^2 = 0, \\
[B, \gamma] &= 0, \quad [c, \gamma] = 0, \\
dK &= 0, \quad dB = K, \\
dc &= cKc - \gamma^2, \\
d\gamma &= cK\gamma - \frac{1}{2}\gamma Kc - \frac{1}{2}\gamma cK, \\
d\gamma^2 &= cK\gamma^2 - \gamma^2 Kc,
\end{aligned}
\tag{1.3}
$$

Notations

$$\begin{aligned} \zeta_0' &= FcKBcF + FB\gamma^2 F, \\ \xi_0' &= FcKB\gamma F + \frac{1}{2} FB\gamma KcF + \frac{1}{2} FB\gamma cKF, \\ \zeta_n' &= \psi_n' + \chi_n', \quad n > 0, \\ \xi_n' &= \vartheta_n' + \eta_n', \quad n > 0, \end{aligned} \tag{1.4}$$

$$\begin{aligned} \psi_n' &= Fc\Omega^n KBcF, \quad n > 0, \\ \chi_n' &= F\gamma\Omega^n KB\gamma F, \quad n > 0, \\ \vartheta_n' &= F\gamma\Omega^n KBcF, \quad n > 0, \\ \eta_n' &= Fc\Omega^n KB\gamma F, \quad n > 0. \end{aligned} \tag{1.5}$$

AGM solution

- Pure-gauge solution for $\lambda < 1$

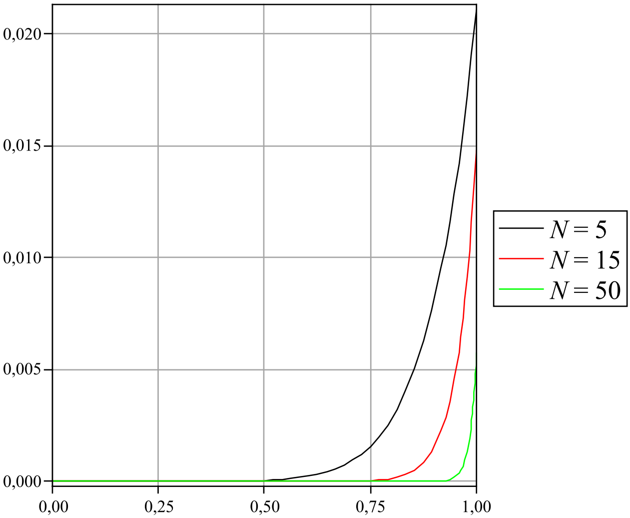
$$\Phi_{+,N}(\lambda) = \sum_{n=0}^{N} \lambda^{n+1}\zeta'_n, \qquad \Phi_{-,N}(\lambda) = \sum_{n=0}^{N} \lambda^{n+1}\xi'_n; \tag{1.6}$$

- Regularized non pure-gauge solution for $\lambda = 1$

$$\Phi^R_{+,N} = \sum_{n=0}^{N} \zeta'_n - \zeta_N - \frac{1}{2}\zeta'_N, \qquad \Phi^R_{-,N} = \sum_{n=0}^{N} \xi'_n - \xi_N - \frac{1}{2}\xi'_N. \tag{1.7}$$

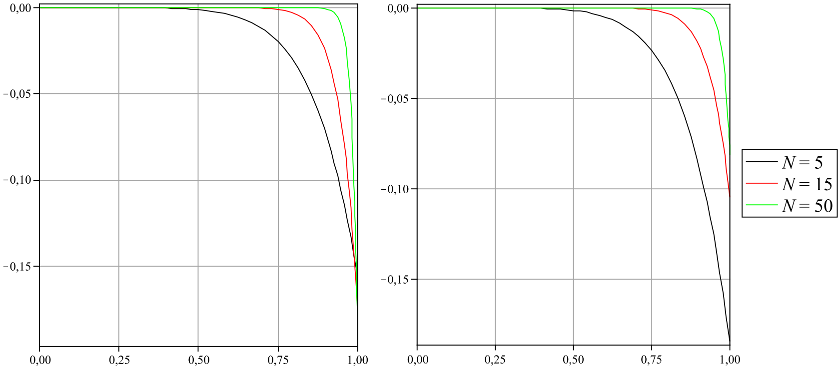
- hep-th/0804.2017

Contraction of equations of motion for $\Phi_{+,N}(\lambda)$, $N = 5, 10, 15, 20, 50$ with ζ_0'



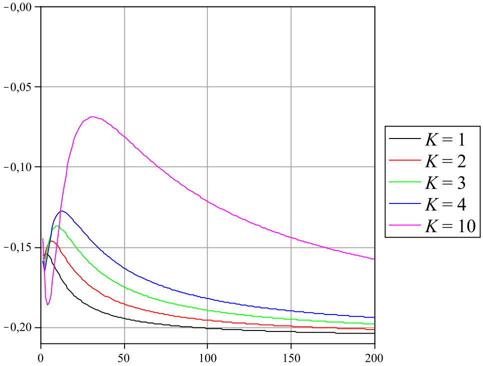
hep-th/0901.4533

Contraction of equations of motion for $\Phi_{+,N}(\lambda)$, $N = 5, 10, 15, 20, 50$ with ψ_1 and ψ_{10}



Necessity of phantom terms

Contraction of equations of motion for $\Phi_{+,N}(\lambda = 1)$ with ψ_K for $K = 1, 2, 3, 4, 10$



K	1	2	10
contraction for $\Phi_{+,1000}(\lambda = 1)$	-0.20649	-0.20594	-0.19605

Finding phantom terms

- Solution with "unknown" phantom terms

$$\Phi^R_{+,N}(a,b) = \sum_{n=0}^{N} \zeta'_n + a\zeta_N + b\zeta'_N,$$
$$\Phi^R_{-,N}(c,d) = \sum_{n=0}^{N} \xi'_n + c\xi_N + d\xi'_N. \qquad (2.8)$$

- We contract E.O.M. for these configurations with different states

$$\begin{cases} \text{contraction with } \zeta'_0 & = 0, \\ \text{contraction with } \psi_1 & = 0. \end{cases} \qquad (2.9)$$

Finding phantom terms - results

- $GSO(+)$

N	a	b
1	-1.009276758	-0.4947687297
5	-1.002887587	-0.4802195028
10	-1.001361647	-0.4839491795
50	-0.999973303	-0.5013822672
100	-0.999996393	-0.5003673241

- $GSO(-)$

N	c	d
1	-1.021127911	-0.4547206603
5	-0.9958034182	-0.5258588923
10	-0.9996956796	-0.5033484993
50	-0.9999993154	-0.5000348047
100	-0.9999999396	-0.5000061487

- Exact values are $a = c = -1$ and $b = d = -\frac{1}{2}$.

Calculation of action

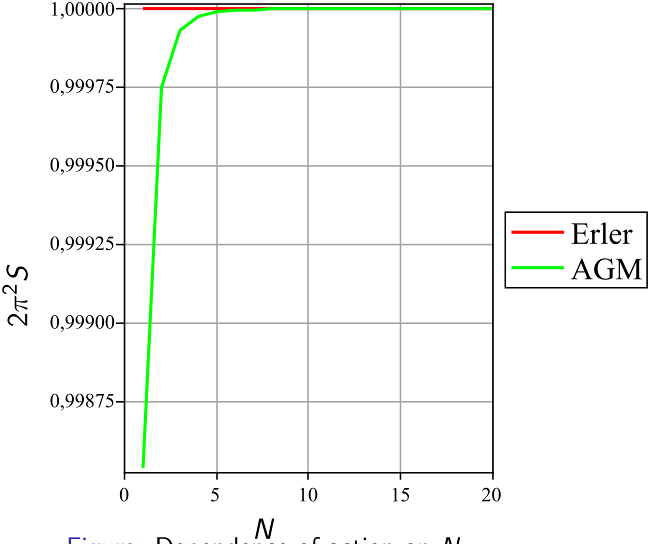


Figure: Dependence of action on N.

Calculation of action

N	$2\pi^2 S(\Phi_{+,N}, \Phi_{-,N})$
1	0.998538383
5	0.999990263
10	0.999999488
15	0.999999919
20	0.999999979
30	0.999999997

Schnabl's solution(2005)

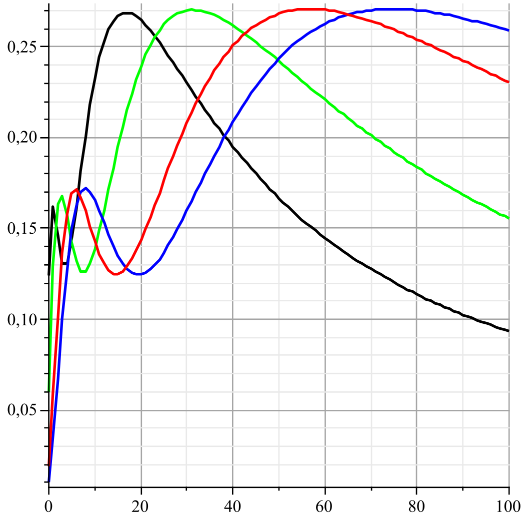


Figure: Residual of equations of motion for Φ_N contracted with φ_K depending on N for $K = 4, 8, 15, 20$