LAGRANGIANS FOR p-ADIC SECTOR OF OPEN SCALAR STRINGS

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1. Introduction

2. p-Adic and Adelic Strings

3. Lagrangian for p-Adic Strings

4. Lagrangians for p-Adic Sector

5. Concluding Remarks

1. Introduction

Lagrangian and EOM for p-Adic Open String:

$$\mathcal{L} = \frac{m^D}{g^2}\frac{p^2}{p-1}\Big[-\frac{1}{2}\varphi\,\mathbf{p}^{-\frac{\Box}{2m^2}}\varphi + \frac{1}{p+1}\varphi^{\mathbf{p}+1}\Big]$$

$$\mathbf{p}^{-\frac{\Box}{2m^2}}\varphi = \varphi^{\mathbf{p}}$$

Lagrangian and EOM for p-Adic Sector of Open Strings:

$$\mathcal{L} = \frac{1}{2}\phi\,\Big[\zeta^{-1}\Big(\frac{\Box}{2}-1\Big) + \zeta^{-1}\Big(\frac{\Box}{2}\Big)\Big]\,\phi - \phi^{2}\,\Phi(\phi)\,,$$

$$\Big[\zeta^{-1}\Big(\frac{\Box}{2}-1\Big) + \zeta^{-1}\Big(\frac{\Box}{2}\Big)\Big]\,\phi = 2\phi\,\Phi(\phi) + \phi^{2}\,\Phi'(\phi)$$

p-Adic numbers:

- discovered by K. Hansel in 1897.

- many applications in mathematics, e.g. representation theory, algebraic geometry and modern number theory.

- many applications in mathematical physics since 1987, e.g. string theory, QFT, quantum mechanics, dynamical systems, ...

- p-adic mathematical physics: (1) Fourth Int. Conf. on p-adic Math. Physics,(Grodno, Belarus, Sept. 2009); (2) int. journal: p-Adic Numbers, Ultrametric Analysis and Applications (Pleiades/Springer).

- Any p-adic number ($x \in \mathbb{Q}_p$) has a unique canonical representation

$$x = p^{\nu(x)} \sum_{n=0}^{+\infty} x_n \, p^n \,, \;\; \nu(x) \in \mathbb{Z} \,, \;\; x_n \in \{0, \, 1, \, \cdots, \, p-1\}.$$

- Real and p-adic numbers unify by adeles. An adele α is an infinite sequence

$$\alpha = (\alpha_\infty, \, \alpha_2, \, \alpha_3, \, \cdots, \, \alpha_p \,, \, \cdots) \,, \qquad \alpha_\infty \in \mathbb{R} \,, \;\; \alpha_p \in \mathbb{Q}_p$$

  where for all but a finite set $\mathcal{P}$ of primes p one has that $\alpha_p \in \mathbb{Z}_p = \{x \in \mathbb{Q}_p \, : |x|_p \leq 1\}$.

2. p-Adic and Adelic Strings

Volovich, Vladimirov, Freund, Witten, Arefeva, B.D., ...

String amplitudes:

- standard crossing symmetric Veneziano amplitude

$$A_\infty(a,b) = g_\infty^2 \int_{\mathbb{R}} |x|_\infty^{a-1} |1-x|_\infty^{b-1} d_\infty x$$

$$= g_\infty^2 \frac{\zeta(1-a)}{\zeta(a)} \frac{\zeta(1-b)}{\zeta(b)} \frac{\zeta(1-c)}{\zeta(c)}$$

- p-adic crossing symmetric Veneziano amplitude

$$A_p(a,b) = g_p^2 \int_{\mathbb{Q}_p} |x|_p^{a-1} |1-x|_p^{b-1} d_p x$$

$$= g_p^2 \frac{1-p^{a-1}}{1-p^{-a}} \frac{1-p^{b-1}}{1-p^{-b}} \frac{1-p^{c-1}}{1-p^{-c}}$$

  where $a,b,c \in \mathbb{C}$ with condition $a+b+c=1$ and $\zeta(a)$ is the Riemann zeta function.

- product formula for adelic strings

$$A(a,b) = A_\infty(a,b) \prod_p A_p(a,b) = g_\infty^2 \prod_p g_p^2 = \text{const.}$$

3. Effective Lagrangian for p-Adic Strings

- One of the greatest achievements in p-adic string theory is an effective field description of scalar open and closed p-adic strings. The corresponding Lagrangians are very simple and exact. They describe not only four-point scattering amplitudes but also all higher (Koba-Nielsen) ones at the tree-level.

- The exact tree-level Lagrangian for effective scalar field φ which describes open p-adic string tachyon is

$$\mathcal{L}_{\rm p} = \frac{m_p^D}{g_p^2}\,\frac{p^2}{p-1}\Big[-\frac{1}{2}\,\varphi\,{\rm p}^{-\frac{\Box}{2m_p^2}}\,\varphi + \frac{1}{p+1}\,\varphi^{\rm p+1}\Big]\,,$$

  where p is any prime number, $\Box = -\partial_t^2 + \nabla^2$ is the D-dimensional d'Alembertian and metric with signature $(- + ... +)$ (Freund, Witten, Frampton, Okada, ...) .

- An infinite number of spacetime derivatives follows from

$${\rm p}^{-\frac{\Box}{2m_p}} = \exp\big(-\frac{\ln p}{2m_p}\Box\big) = \sum_{\rm k\geq 0}\big(-\frac{\ln p}{2m_p}\big)^{\rm k}\,\frac{1}{k!}\,\Box^{\rm k}\,.$$

- The equation of motion is

$$\mathbf{p}^{-\frac{\Box}{2m_p^2}}\varphi = \varphi^{\mathbf{p}},$$

and its properties have been studied by many authors (see e.g. Vladimirov, Barnaby and references therein). It has trivial solutions $\varphi = 0$ and $\varphi = 1$. There are also inhomogeneous solutions resembling solitons. Equation separates in arguments and for any spatial direction x^i one has

$$\varphi(\mathbf{x^i}) = \mathbf{p}^{\frac{1}{2(p-1)}} \exp\big(-\frac{p-1}{2m_p^2\, p \ln p}(\mathbf{x^i})^2\big).$$

- Prime number p can be replaced by natural number $n \geq 2$ and such expression also makes sense. Moreover, when $p = 1 + \varepsilon \to 1$ there is the limit which is related to the ordinary bosonic string in the boundary string field theory (Gerasimov-Shatashvili):

$$\mathcal{L} = \frac{m^D}{g^2}\big[\frac{1}{2}\varphi\frac{\Box}{m^2}\varphi + \frac{\varphi^2}{2}(\ln\varphi^2 - 1)\big].$$

- **This p-adic string theory has been significantly pushed forward when was shown (Ghoshal-Sen) that it describes tachyon condensation and brane descent relations.**

- **After that success, many aspects of p-adic string dynamics have been investigated and compared with dynamics of ordinary strings (see, e.g. Minahan, Zwiebach, Moeller, ...).**

- **Noncommutative deformation by the Moyal star product and of p-adic string world-sheet with a constant B-field was investigated (Ghoshal, ...).**

- **A systematic mathematical study of spatially homogeneous solutions of the nonlinear equation of motion (Vladimirov).**

- **Some possible cosmological implications of p-adic string theory have been also investigated (Arefeva, Barnaby, Joukovskaya, ...).**

- **It was proposed (Ghoshal) that p-adic string theories provide lattice discretization to the world-sheet of ordinary strings.**

- **As a result of these developments it follows that many nontrivial features of ordinary strings are similar to p-adic ones and are related to the p-adic effective action.**

4. Lagrangians for p-Adic Sector

A. Additive approach

B.D.: hep-th/0703008v1, 0804.4114v1[hep-th],

0805.0403v1[hep-th], 0809.1601[hep-th]

- **Now we want to introduce a model which incorporates all the above n-adic string Lagrangians, so to have the Riemann zeta function. We take the sum of the Lagrangians $\mathcal{L}_n$ in the form**

$$\mathbf{L} = \sum_{\mathbf{n}\geq 1} \mathbf{C_n}\, \mathcal{L}_\mathbf{n}$$

$$\mathcal{L}_n = \sum_{n\geq 1} \frac{m_n^D}{g_n^2} \frac{n^2}{n-1} \Big[-\frac{1}{2}\phi \sum_{n\geq 1} n^{-\frac{\Box}{2m_n^2}} \phi + \sum_{n\geq 1} \frac{1}{n+1} \phi^{n+1} \Big]$$

  which depends on the choice of coefficients C_n, masses m_n and coupling constants g_n^2.

- **There is a few simple and interesting cases ($m_n = m,\ g_n^2 = g^2$):**

$$C_n = \frac{n-1}{n^{2+h}}$$

$$C_n = \frac{n^2-1}{n^2}$$

$$C_n = \mu(n)\, \frac{n-1}{n^2}$$

  where $\mu(n)$ is the Möbius function.

- **Let us consider the first case** $C_n = \frac{n-1}{n^{2+h}}$ **:**

$$\mathbf{L_h} = \frac{1}{g^2}\,[\,-\,\frac{1}{2}\,\phi\sum_{\mathbf{n\geq 1}}\mathbf{n}^{-\frac{\Box}{2m^2}-\mathbf{h}}\,\phi + \sum_{\mathbf{n\geq 1}}\frac{n^{-h}}{n+1}\,\phi^{\mathbf{n+1}}]\,.$$

- According to the famous Euler product formula one can write

$$\sum_{\mathrm{n}\geq 1} \mathrm{n}^{-\frac{\Box}{2m^2}} = \prod_{\mathrm{p}} \frac{1}{1 - p^{-\frac{\Box}{2m^2}}}\,.$$

- Recall that the Riemann zeta function is defined as

$$\zeta(\mathrm{s}) = \sum_{\mathrm{n}\geq 1} \frac{1}{n^s} = \prod_{\mathrm{p}} \frac{1}{1 - p^{-s}}\,, \qquad \mathrm{s} = \sigma + \mathrm{i}\tau\,, \;\; \sigma > 1\,.$$

- We can rewrite Lagrangian in the form

$$\mathrm{L_h} = -\frac{1}{g^2}\Big[\frac{1}{2}\,\phi\,\zeta\Big(\frac{\Box}{2m^2} + \mathrm{h}\Big)\,\phi + \mathcal{AC}\sum_{\mathrm{n}=1}^{+\infty} \frac{n^{-h}}{n+1}\,\phi^{\mathrm{n}+1}\Big]\,.$$

We shall consider this Lagrangian with analytic continuation of zeta functions and power series.

- $\zeta(\frac{\Box}{2m^2})$ acts as a pseudodifferential operator in the following way:

$$\zeta(\frac{\Box}{2m^2})\,\phi(\mathbf{x}) = \frac{1}{(2\pi)^D}\int_{\mathbb{R}^D} e^{\mathbf{ixk}}\,\zeta(-\frac{k^2}{2m^2})\,\tilde{\phi}(\mathbf{k})\,\mathbf{dk}\,,$$

  where $\tilde{\phi}(k) = \int e^{(-ikx)}\,\phi(x)\,dx$ is the Fourier transform of $\phi(x)$.

- Nonlocal dynamics of this field ϕ is encoded in the pseudo-differential form of the Riemann zeta function (the d'Alembertian is an argument of the Riemann zeta function).

- Potential of the above scalar field is equal to $-L_h$ at $\Box = 0$, i.e.

$$V_h(\phi) = \frac{1}{g^2}\,(\frac{\phi^2}{2}\,\zeta(h) - \mathcal{AC}\sum_{n=1}^{+\infty}\frac{n^{-h}}{n+1}\,\phi^{n+1})$$

  where $h \neq 1$ since $\zeta(1) = \infty$. The term with ζ-function vanishes at $h = -2, -4, -6, \cdots$.

- The equation of motion in differential and integral form is

$$\zeta(\frac{\Box}{2\,m^2} + h)\,\phi = \mathcal{AC}\sum_{n=1}^{+\infty} n^{-h}\,\phi^n\,,$$

$$\frac{1}{(2\pi)^D}\int_{\mathbb{R}^D} e^{ixk}\,\zeta(-\frac{k^2}{2\,m^2} + h)\,\tilde{\phi}(k)\,dk = \mathcal{AC}\sum_{n=1}^{+\infty} n^{-h}\,\phi^n\,,$$

  respectively. It is clear that $\phi = 0$ is a trivial solution for any real h. When $h > 1$ we have another constant trivial solution $\phi = 1$.

- **In the weak field approximation** $(|\phi(x)| \ll 1)$ **the above expression becomes**

$$\int_{\mathbb{R}^D} e^{ikx} \, [\zeta(- \frac{k^2}{2\, m^2} + h) - 1] \, \tilde{\phi}(k) \, dk = 0 \, ,$$

which has a solution $\tilde{\phi}(k) \neq 0$ **if equation**

$$\zeta(\frac{-k^2}{2\, m^2} + h) = 1$$

is satisfied. According to the usual relativistic kinematic relation $k^2 = -k_0^2 + \overrightarrow{k}^2 = -M^2$**, equation in the form**

$$\zeta(\frac{M^2}{2\, m^2} + h) = 1 \, ,$$

determines mass spectrum $M^2 = \mu_h \, m^2$**, where set of values of spectral function** μ_h **depends on** h**. The above equation gives infinitely many tachyon mass solutions.**

EXAMPLE (h = 0)

- **The related Lagrangian is**

$$L_0 = -\frac{m^D}{g^2}\Big[\frac{1}{2}\phi\,\zeta\big(\frac{\Box}{2\,m^2}\big)\,\phi + \phi + \frac{1}{2}\ln(1-\phi)^2\Big]\,.$$

- **The corresponding potential is**

$$V_0(\phi) = \frac{m^D}{g^2}\Big[\frac{\zeta(0)}{2}\,\phi^2 + \phi + \frac{1}{2}\ln(1-\phi)^2\Big]\,,$$

where $\zeta(0) = -\frac{1}{2}$. It has two local maxima: $V_0(0) = 0$ and $V_0(3) \approx 1.443\frac{m^D}{g^2}$. There are no stable points and $\lim_{\phi\to 1} V_0(\phi) = -\infty\,,\ \lim_{\phi\to\pm\infty} V_0(\phi) = -\infty\,.$

- **The equation of motion for ϕ is**

$$\zeta\big(\frac{\Box}{2m^2}\big)\,\phi = \frac{1}{(2\pi)^D}\int_{\mathbb{R}^D} e^{ixk}\,\zeta\big(-\frac{k^2}{2m^2}\big)\,\tilde{\phi}(k)\,dk = \frac{\phi}{1-\phi}\,.$$

It has two trivial solutions: $\phi = 0$ and $\phi = 3$. The solution $\phi = 0$ is evident. The solution $\phi = 3$ follows from the Taylor expansion of the Riemann zeta function operator

$$\zeta\big(\frac{\Box}{2m^2}\big) = \zeta(0) + \sum_{n\geq 1}\frac{\zeta^{(n)}(0)}{n!}\big(\frac{\Box}{2m^2}\big)^n\,, \qquad \zeta(0) = -\frac{1}{2}\,.$$

So far nontrivial solutions are unknown.

- **In the weak field approximation**

$$\zeta(\frac{\Box}{2m^2})\,\phi = \frac{1}{(2\pi)^D}\int_{\mathbb{R}^D} e^{ixk}\,\zeta(-\frac{k^2}{2m^2})\,\tilde{\phi}(\mathbf{k})\,d\mathbf{k} = \phi$$

on mass shell one obtains equation for the mass spectrum

$$\zeta(\frac{m^2}{2}) = 1$$

which has infinitely many tachyon solutions.

Case $C_n = \frac{n^2-1}{n^2}$

- **In this case Lagrangian becomes**

$$\mathbf{L} = \frac{m^D}{g^2}[\,-\,\frac{1}{2}\,\phi\,\sum_{\mathbf{n=1}}^{+\infty}(\mathbf{n}^{-\frac{\Box}{2m^2}+1} + \mathbf{n}^{-\frac{\Box}{2m^2}})\,\phi + \sum_{\mathbf{n=1}}^{+\infty}\phi^{\mathbf{n+1}}]$$

and it yields

$$\mathbf{L} = \frac{m^D}{g^2}[\,-\,\frac{1}{2}\,\phi\,\{\zeta(\frac{\Box}{2\,m^2} - \mathbf{1}) + \zeta(\frac{\Box}{2\,m^2})\}\,\phi\, + \,\frac{\phi^2}{1-\phi}]\,.$$

- **The corresponding potential is**

$$\mathbf{V}(\phi) = -\frac{m^D}{g^2}\,\frac{31-7\phi}{24\,(1-\phi)}\,\phi^{\mathbf{2}}\,,$$

which has the following properties: $V(0) = V(31/7) = 0\,,\; V(1\pm 0) = \pm\infty\,,\; V(\pm\infty) = -\infty$. **At** $\phi = 0$ **potential has local maximum.**

- **The equation of motion is**

$$[\zeta(\frac{\Box}{2\,m^2} - \mathbf{1}) + \zeta(\frac{\Box}{2\,m^2})]\,\phi = \frac{\phi((\phi-1)^2+1)}{(\phi-1)^2}\,,$$

which has only $\phi = 0$ **as a constant real solution.**

Case $C_n = \mu(n)\frac{n-1}{n^2}$

$$\mu(n) = \begin{cases} 0, & n = p^2 m \\ (-1)^k, & n = p_1 p_2 \cdots p_k,\ p_i \neq p_j \\ 1, & n = 1,\ (k = 0)\,. \end{cases} \qquad (1)$$

- **In this case Lagrangian becomes**

$$\mathbf{L}_\mu = \frac{m^D}{g^2}\Big[-\frac{1}{2}\,\phi \sum_{\mathbf{n}=\mathbf{1}}^{+\infty} \frac{\mu(n)}{n^{\frac{\Box}{2m^2}}}\,\phi + \sum_{\mathbf{n}=\mathbf{1}}^{+\infty} \frac{\mu(n)}{n+1}\,\phi^{\mathbf{n}+\mathbf{1}}\Big]$$

and it yields

$$\mathbf{L}_\mu = \frac{m^D}{g^2}\Big[-\frac{1}{2}\,\phi\, \frac{1}{\zeta(\frac{\Box}{2m^2})}\,\phi + \int_{\mathbf{0}}^{\phi} \mathcal{M}(\phi)\, \mathbf{d}\phi\Big]\,,$$

where $\mathcal{M}(\phi) = \sum_{n=1}^{+\infty} \mu(n)\, \phi^n = \phi - \phi^2 - \phi^3 - \phi^5 + \phi^6 - \phi^7 + \phi^{10} - \phi^{11} - \ldots$.

- **The corresponding potential is**

$$\mathbf{V}_\mu(\phi) = -\mathbf{L}_\mu(\Box = \mathbf{0}) = -\frac{m^D}{g^2}\Big[\phi^{\mathbf{2}} + \int_{\mathbf{0}}^{\phi} \mathcal{M}(\phi)\, \mathbf{d}\phi\Big]\,.$$

- **The equation of motion is**

$$\frac{1}{\zeta(\frac{\Box}{2m^2})}\,\phi - \mathcal{M}(\phi) = \mathbf{0}$$

which has $\phi = 0$ as a constant real solution.

- **Its weak field approximation implies condition on the mass spectrum**

$$\frac{1}{\zeta(\frac{M^2}{2m^2})} - 1 = 0.$$

B. Multiplicative approach

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Let us now consider a new approach, which is not based on a summation of p-adic Lagrangians, but the Riemann zeta function will emerge through its product form. Our starting point is again p-adic Lagrangian with equal masses, i.e. $m_p^2 = m^2$ for every p. It is useful to rewrite Lagrangian, first in the form,

$$\mathcal{L}_\mathbf{p} = \frac{m^D}{g_p^2}\frac{p^2}{p^2-1}\{-\frac{1}{2}\varphi\,[\mathbf{p}^{-\frac{\Box}{2m^2}+\mathbf{1}} + \mathbf{p}^{-\frac{\Box}{2m^2}}]\,\varphi + \varphi^{\mathbf{p}+\mathbf{1}}\}$$

and then, by addition and substraction of φ^2, as

$$\mathcal{L}_\mathbf{p} = \frac{m^D}{g_p^2}\frac{p^2}{p^2-1}\{\frac{1}{2}\varphi\,[(\mathbf{1}-\mathbf{p}^{-\frac{\Box}{2m^2}+\mathbf{1}})$$

$$+(\mathbf{1}-\mathbf{p}^{-\frac{\Box}{2m^2}})]\,\varphi - \varphi^{\mathbf{2}}(\mathbf{1}-\varphi^{\mathbf{p}-\mathbf{1}})\}\,.$$

Now we introduce a Lagrangian for the entire p-adic sector by taking products

$$\prod_{\mathbf{p}} \mathbf{g}_{\mathbf{p}}^{\mathbf{2}} = \mathbf{C}\,, \quad \prod_{\mathbf{p}} \frac{1}{1 - p^{-2}}\,, \quad \prod_{\mathbf{p}} (\mathbf{1} - \mathbf{p}^{-\frac{\Box}{2m^2}+\mathbf{1}})\,,$$

$$\prod_{\mathbf{p}} (\mathbf{1} - \mathbf{p}^{-\frac{\Box}{2m^2}}) \quad \prod_{\mathbf{p}} (\mathbf{1} - \varphi^{\mathbf{p}-\mathbf{1}})$$

at the corresponding places. Then this new Lagrangian becomes

$$\mathcal{L} = \frac{m^D}{C}\,\zeta(\mathbf{2})\,\{\frac{1}{2}\,\phi[\zeta^{-\mathbf{1}}(\frac{\Box}{2m^2} - \mathbf{1})$$

$$+\zeta^{-\mathbf{1}}(\frac{\Box}{2m^2})]\,\phi - \phi^{\mathbf{2}}\prod_{\mathbf{p}}(\mathbf{1} - \phi^{\mathbf{p}-\mathbf{1}})\}\,,$$

where $\zeta^{-1}(s) = 1/\zeta(s)$ and new scalar field is denoted by ϕ. For the coupling constant g_p there are two interesting possibilities: (1) $g_p^2 = \frac{p^2}{p^2-1}$, what yields $\zeta(2)/C = 1$, and (2) $g_p = |r|_p$, where r may be any non zero rational number and it gives

$|r|_\infty \prod_p |r|_p = 1$ (this possibility was already considered by Dragovich). Both these possibilities are consistent with adelic product formula. For simplicity, in the sequel we shall take $C = \zeta(2)$. It is worth noting that having Lagrangian one can easily reproduce its p-adic ingredient.

Let us rewrite the above Lagrangian in the simple form

$$\mathcal{L} = \frac{1}{2}\phi\,[\zeta^{-1}(\frac{\Box}{2} - \mathbf{1}) + \zeta^{-1}(\frac{\Box}{2})]\,\phi - \phi^{\mathbf{2}}\,\mathbf{\Phi}(\phi)\,,$$

with $m = 1$ and $\Phi(\phi) = \mathcal{AC}\prod_p(1 - \phi^{p-1})$, where $\mathcal{AC}$ denotes analytic continuation of infinite product $\prod_p(1 - \phi^{p-1})$, which is convergent if $|\phi|_\infty < 1$. One can easily see that $\Phi(0) = 1$ and $\Phi(1) = \Phi(-1) = 0$.

The corresponding equation of motion is

$$[\zeta^{-1}(\frac{\Box}{2} - \mathbf{1}) + \zeta^{-1}(\frac{\Box}{2})]\,\phi = \mathbf{2}\phi\,\mathbf{\Phi}(\phi) + \phi^{\mathbf{2}}\,\mathbf{\Phi}'(\phi)\,,$$

and has $\phi = 0$ as a trivial solution. In the weak-field approximation ($\phi(x) \ll 1$), equation becomes

$$[\zeta^{-1}(\frac{\Box}{2} - \mathbf{1}) + \zeta^{-1}(\frac{\Box}{2})]\,\phi = \mathbf{2}\phi\,.$$

Note that the above operator-valued zeta function can be regarded as a pseudodifferential operator.

Mass spectrum of M^2 is determined by solutions of equation

$$\zeta^{-1}(\frac{M^2}{2} - 1) + \zeta^{-1}(\frac{M^2}{2}) = 2\,.$$

There are infinitely many tachyon solutions, which are below largest one $M^2 \approx -3.5$.

The potential follows from $-\mathcal{L}$ at $\Box = 0$, i.e.

$$\mathbf{V}(\phi) = [7 + \mathbf{\Phi}(\phi)]\,\phi^2\,,$$

since $\zeta(-1) = -1/12$ and $\zeta(0) = -1/2$. This potential has local minimum $V(0) = 0$ and values $V(\pm 1) = 7$. To explore behavior of $V(\phi)$ for all $\phi \in \mathbb{R}$ one has first to investigate properties of the function $\Phi(\phi)$.

It is worth noting that a Lagrangian similar to the above one can be obtained by an additive approach. Namely, let us start from $g_p^2 = p^2/(p^2-1)$ and $m=1$, we have

$$\mathbf{L} = -\sum_{\mathbf{n}=1}^{+\infty} \mu(\mathbf{n})\,\mathcal{L}_{\mathbf{n}} = \frac{1}{2}\,\phi\,[\sum_{\mathbf{n}=1}^{+\infty} \mu(\mathbf{n})\,\mathbf{p}^{-\frac{\Box}{2}+1}$$

$$+\sum_{\mathbf{n}=1}^{+\infty} \mu(\mathbf{n})\,\mathbf{p}^{-\frac{\Box}{2}}]\,\phi - \sum_{\mathbf{n}=1}^{+\infty} \mu(\mathbf{n})\,\phi^{\mathbf{n}+1}\,,$$

where $\mu(n)$ is the above Möbius function. Introducing zeta function one can rewrite it in the form

$$\mathbf{L} = \frac{1}{2}\,\phi\,[\zeta^{-1}(\frac{\Box}{2} - 1)$$

$$+\zeta^{-1}(\frac{\Box}{2})]\,\phi - \phi^{2}\,\mathbf{F}(\phi)\,,$$

where $F(\phi) = \sum_{n=1}^{+\infty} \mu(n)\,\phi^{n-1}$. The difference between two Lagrangians is only in functions $\Phi(\phi)$ and $F(\phi)$. Since $\Phi(\phi) = (1-\phi)(1-\phi^2)(1-\phi^4)... = 1-\phi-\phi^2+\phi^3-\phi^4+...$ and $F(\phi) = 1-\phi-\phi^2-\phi^4+...$, it follows that these functions have the same behavior for $|\phi| \ll 1$. Hence, Lagrangians have the same mass spectrum and in weak-field approximation describe the same scalar field theory.

5. Concluding Remarks

- p-Adic strings are in many ways related to ordinary strings. It seems that ordinary and p-adic strings are different faces of an adelic string.
- p-Adic scalar tachyons at the tree level are described by simple and exact nonlocal Lagrangian.
- All here constructed Lagrangians contain Riemann zeta function nonlocality.
- These Lagrangians with ζ-function nonlocality are new and significant in their own right.
- Find the corresponding string amplitudes.
- Investigate possible cosmological aspects.

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