Holographic Conformal Structure of the Kerr-Schild Black-Hole Geometry and Twistor-Beam Solutions.

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Kerr-Schild form of the rotating black hole solutions:

$$g_{\mu\nu} = \eta_{\mu\nu} + 2Hk_\mu k_\nu, \qquad H = \frac{mr - e^2/2}{r^2 + a^2\cos^2\theta}. \tag{1}$$

Vector field $k_\mu(x)$ is tangent to Principal Null Congruence (PNC).

$$k_\mu(x) = P^{-1}(du + \bar{Y}d\zeta + Yd\bar{\zeta} - Y\bar{Y}dv), \tag{2}$$

where $Y(x) = e^{i\phi}\tan\frac{\theta}{2}$, and the null Cartesian coordinates are

$$2^{\frac{1}{2}}\zeta = x + iy,\ 2^{\frac{1}{2}}\bar{\zeta} = x - iy,\ 2^{\frac{1}{2}}u = z - t,\ 2^{\frac{1}{2}}v = z + t. \tag{3}$$

Congruence of twistors PNC is controlled by *Kerr Theorem.*

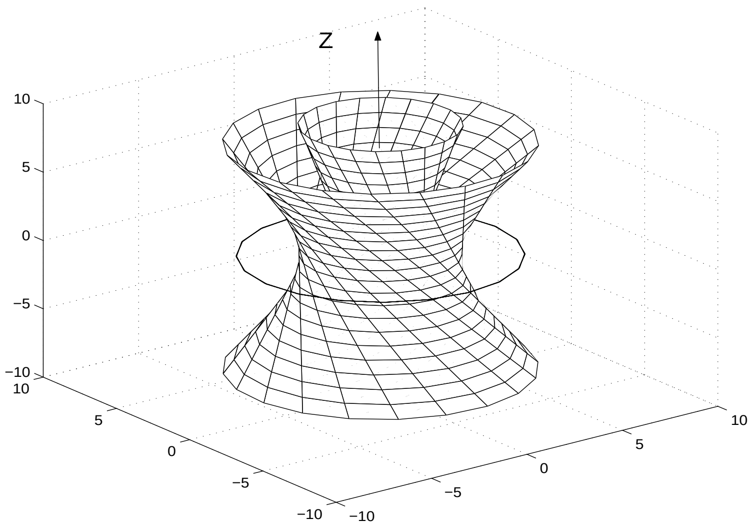

Figure 1: The Kerr singular ring and the Kerr congruence.

The Kerr singular ring $r = \cos\theta = 0$ is a branch line of space on two sheets: "negative (–)" and "positive (+)" where the fields change their directions. In particular,

$$k^{\mu(+)} \neq k^{\mu(-)} \quad \Rightarrow \qquad g^{(+)}_{\mu\nu} \neq g^{(-)}_{\mu\nu}. \tag{4}$$

Twosheetedness! Mystery of the Kerr source!
Oblate spheroidal coord. $x + iy = (r + ia)e^{i\phi} \sin\theta, \quad z = r\cos\theta$, cover spacetime twice. Disk $r = 0$ separates the 'out'-sheet $r > 0$, from 'in'-sheet $r < 0$.
a) Closed string: E.Newman&A.Janis (JMP1964), AB (JETP1974, PRD03), W.Israel(PRD1975). 'Alice string' akin to the Sen heterotic string solution to low-energy string theory, AB(PRD1995).
b) Rotating superconducting disk. W.Israel (PRD 1970), Hamity, I.Tiomno (1973), C.A. L'opez (PRD 1983)9; A.B. (1989,2000-2004), The Kerr ring as a 'mirror gate' to 'Alice' world,.
New Look: Holographic interpretation. (AB 0903.2365[hep-th]) based on the ideas C.R. Stephens, G. t' Hooft and B.F. Whiting (1994), 't Hooft (2000), Bousso (2002).

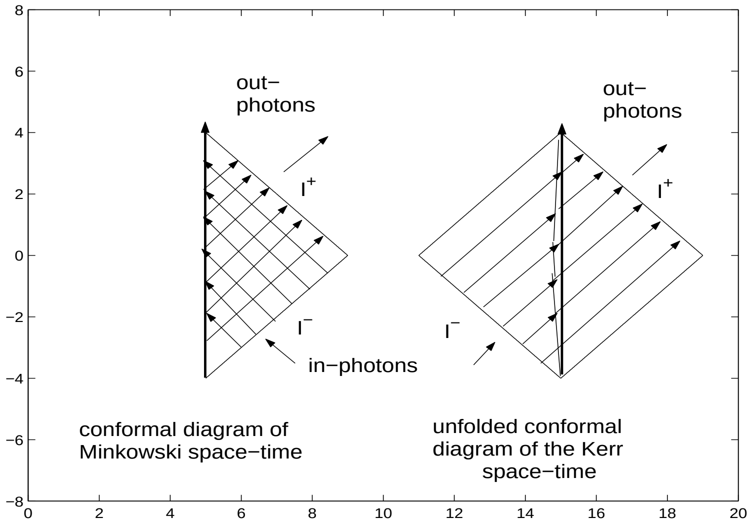

Figure 2: Penrose conformal diagrams. The unfolded Kerr-Schild spacetime corresponds to the holographic structure of a quantum black-hole spacetime.

Twosheetedness of the Kerr-Schild geometry corresponds to holographic black-hole KS spacetime. For pure gravity BH solutions, the both sheets can be used as 'physical' ones.
Alignment of the electromagnetic field to PNC and gravitational field, $A_\mu k^\mu = 0$! $\Rightarrow$ PNC is in-going at $r < 0$, passing through the Kerr ring to 'positive' sheet, $r > 0$, and turns into out-going. For $m > 0$ the horizon exists only for $r > 0$ $\Rightarrow$ only out-sheet may be identified as a 'physical sheet' of the BH.
Stephens, t' Hooft and Whiting (1994) predicted desirable structure of a quantum BH spacetime: the in- and out-sheets have to be separated by a (holographically dual) boundary.
Kerr congruence performs holographic projection of 3+1 dim bulk to 2+1 dim boundary: $Y \in S^2$. "Geodesic and shear-free" (GSF) congruences: $GSF \quad \Leftrightarrow \quad Y,_2 = Y,_4 = 0$ provide extension of the conformal structure from boundary S^2 to bulk.
Kerr Theorem controls the GSF null congruences

$$k_\mu(x) = P^{-1}(du + \bar{Y} d\zeta + Y d\bar{\zeta} - Y\bar{Y} dv). \tag{5}$$

Any GSF congruence is determined by a holomorphic function $Y(x)$ which is solution of equation $F(T^a) = 0$, where F is an arbitrary analytic function of the projective twistor coordinates $T^a = \{Y, \quad \zeta - Yv, \quad u + Y\bar{\zeta}\}$.

Projective spinor $Y = \pi^1/\pi^2$ *determines null direction* k^μ.
Twistor $\equiv \{x^\mu, \pi^a\}$, **or** $Z^A = \{\pi^a, \mu_{\dot{a}}\}$, where $\mu_{\dot{a}} = x_\nu \sigma^\nu_{\dot{a}a} \pi^a$.

The exact stationary KS solutions Debney, Kerr and Schild (1969). A black-hole at rest: $g_{\mu\nu} = \eta_{\mu\nu} + 2Hk_\mu k_\nu, \;\; P = 2^{-1/2}(1 + Y\bar{Y})$.
Tetrad components of electromagnetic field $\mathcal{F}_{ab} = e^\mu_a e^\nu_b \mathcal{F}_{\mu\nu}$,

$$\mathcal{F}_{12} = AZ^2, \quad \mathcal{F}_{31} = \gamma Z - (AZ)_{,1}\,, \tag{6}$$

here $Z = -P/(r + ia\cos\theta)$ is a complex expansion of the congruence. Stationarity $\Rightarrow \gamma = 0$.
The Kerr-Schild form of metric $g^{\mu\nu} = \eta^{\mu\nu} - 2Hk^\mu k^\nu$, where

$$H = \frac{mr - |\psi|^2/2}{r^2 + a^2\cos^2\theta}, \tag{7}$$

and A has the general form

$$A = \psi(Y)/P^2, \tag{8}$$

where $\psi(Y)$ is an arbitrary holomorphic function of $Y(x) = e^{i\phi}\tan\frac{\theta}{2}$, $Y \in S^2$. Contrary to perturbative approach **no smooth harmonic solutions!**

Kerr-Newman solution is exclusive: $\psi(Y) = const.$
In general case there is an infinite set of the exact solutions, in which $\psi(Y)$ is singular at the set of points $\{Y_i, i = 1, 2, ...\}$, $\psi(Y) = \sum_i \frac{q_i}{Y(x) - Y_i}$ corresponding to angular directions $\phi_i,\ \theta_i$.

Twistor-beams. Poles at Y_i produce half-strings: singular *lightlike beams*, supported by twistor rays of the Kerr congruence. The twistor-beams turn in the far zone into string-like exact pp-wave solutions (A.Peres).

How act such beams on the BH horizon?

Black holes with holes in the horizon A.B., E.Elizalde, S.R.Hildebrandt and G.Magli, Phys. Rev. **D74** (2006) 021502(R)

Singular beams lead to formation of the holes in the black hole horizon, which opens up the interior of the "black hole" to external space.

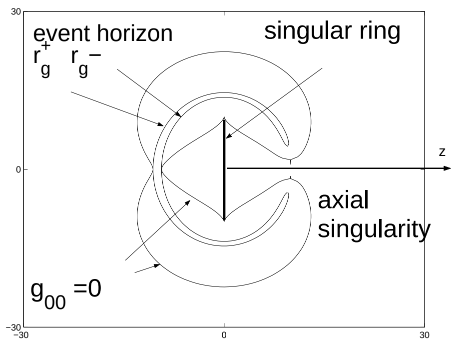

Figure 3: Near extremal black hole with a hole in the horizon, caused by by lightlike singular beam. The event horizon is a closed surface surrounded by surface $g_{00} = 0$.

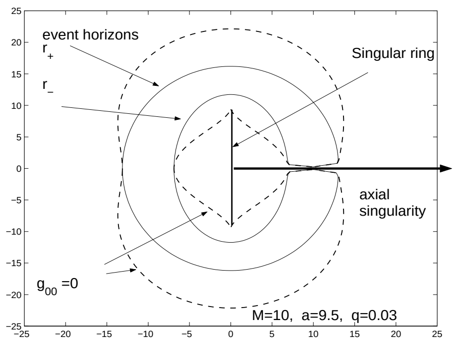

Figure 4: Singular beam forms a small hole in the horizon.

Time-dependent solutions of DKS equations for electromagnetic excitations, $\gamma \neq 0$, A.B. (2004-2008)
a) Exact solutions for electromagnetic field on the Kerr-Schild background, (2004),
b) Asymptotically exact wave solutions, consistent with Kerr-Schild gravity in the low frequency limit, (2006)
c) Exact solutions, consistent with gravity for regularized and averaged stress-energy tensor, (2008)

Electromagnetic field is determined by functions A and γ,

$$A,_2 - 2Z^{-1}\bar{Z}Y,_3 A = 0, \quad A,_4 = 0, \tag{9}$$

$$\mathcal{D}A + \bar{Z}^{-1}\gamma,_2 - Z^{-1}Y,_3 \gamma = 0, \tag{10}$$

and

Gravitational sector: has two equations for function M, which take into account the action of electromagnetic field

$$M,_2 - 3Z^{-1}\bar{Z}Y,_3 M = A\bar{\gamma}\bar{Z}, \tag{11}$$

$$\mathcal{D}M = \frac{1}{2}\gamma\bar{\gamma}. \tag{12}$$

Operator $\mathcal{D}$ is

$$\mathcal{D} = \partial_3 - Z^{-1}Y,_3 \partial_1 - \bar{Z}^{-1}\bar{Y},_3 \partial_2 \ . \tag{13}$$

Similar to the exact stationary solutions, there are **no exact wave solutions with smooth angular dependence!** *Typical solutions contain outgoing singular beam pulses which have very strong back reaction to metric and perforate horizon.*

Treatment of the back reaction of the vacuum fluctuations on the horizon and metric was initiated by J.York (1983) with spherical zero-point excitations (quasi-normal modes) and then Parikh and Wilczek (2001): spherical deformation of the metric and horizon. It was also supported by loop quantum gravity (C. Rovelli, 1996) and Jiang, S. Wu, and X. Cai (2006), J. Zhang, and Z. Zhao (2006) .... However, only the simplest deformations of the horizon, caused by spherical or ellipsoidal quasi-normal modes.
Twistor-beams. Exact stationary and time-dependent Kerr-Schild solutions show that 'elementary' electromagnetic excitations have *singular beams* supported by twistor lines. Interaction of a black-hole with electromagnetic vacuum fluctuations resulted in a fine-grained structure of the horizon pierced by fluctuating microholes.

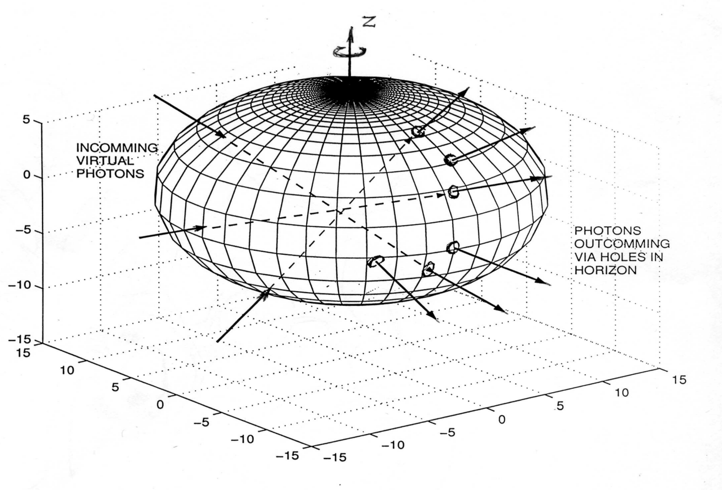

Figure 5: Excitations of a black hole by electromagnetic vacuum creates twistor-beams creating a horizon covered by fluctuating micro-holes.

Solutions of electromagnetic sector.

GSF condition: $Y,_2 = Y,_4 = 0, \Rightarrow$ alignment $k^\mu \partial_\mu Y = 0 \quad \Rightarrow$ exact stationary Kerr-Schild solutions (DKS 1969)

$A = \psi P^{-2}$, where $\psi,_2 = \psi,_4 = 0 \Rightarrow \psi(Y) \Rightarrow k^\mu \partial_\mu \psi = 0.$

Time-dependent solutions (A.B. 2004):

a complex retarded time τ, alignment condition: $\tau,_2 = \tau,_4 = 0$ $\Rightarrow$ extra term $\gamma(Y,\tau)$ caused by time-dependence of $\psi(Y,\tau)$. Integration yields

$$\gamma = \frac{2^{1/2}\dot{\psi}}{P^2 Y} + \phi(Y,\tau)/P, \tag{14}$$

and shows that *time-dependence,* $\dot{\psi} = \sum_i \dot{c}_i(\tau)/(Y - Y_i) \neq 0$, *creates generally the poles in* $\gamma \sim \sum_i q_i/(Y - Y_i)$, *leading to twistor-beams in directions* $Y_i = e^{i\phi_i} \tan\frac{\theta_i}{2}$.

Regularization.

Tuning of the free function $\phi(Y,\tau)$

$$\phi_{tuned}(Y,\tau) = \frac{1}{Y P_i(Y)}(c_i(\tau)\Phi_n(Y) - \frac{1}{Y - Y_i}), \tag{15}$$

where $P_i(Y) = (1 + Y\bar{Y}_i)/\sqrt{2}$ and $\Phi_n(Y) = \prod_i^n (Y - Y_i)$, allows one to regularize γ by cancelling the poles

$$\gamma_{reg} = \gamma_\psi - \phi_{tuned}/P = -\sum_i \frac{c_i(Y - Y_i)\Phi_n(Y)}{Y P^2} \tag{16}$$

and to get stress-energy tensor leading to the exact and consistent with KS gravity solutions in the low-frequency limit.

Quantum gravity approach suggested by B.DeWitt:
1) Solutions of the Maxwell eqs. on the curved background $\Rightarrow T^{\mu\nu}$,
2) Quantization of the electromagnetic field and regularization of stress-energy tensor: $T^{\mu\nu}_{reg} = T^{\mu\nu} - < 0|T^{\mu\nu}|0 >$,
3) Classical Einstein equations for the regularized rhs.

$$R^{\mu\nu} - \frac{1}{2} R g^{\mu\nu} = -\frac{1}{8\pi} T^{\mu\nu}_{reg}. \tag{17}$$

We obtained (arXiv: 0903.2365; PLB **671** 486(2009)) the classical time-dependent solutions for electromagnetic excitations on the BH background, twistor-beams, and back reaction of the beams to the metric and horizon. The solutions are exact and consistent to the Einstein equations with regularized and averaged stress-energy tensor $R^{\mu\nu} - \frac{1}{2} R g^{\mu\nu} = -\frac{1}{8\pi} < T^{\mu\nu}_{reg} >$

Regularization is also classical and is applied only to term γ which determines $T^{\mu\nu} = P \frac{1}{2} \bar{\gamma} \gamma k^\mu k^\nu$. The term $\psi(Y, \tau)$ is not regularized.

$$g_{\mu\nu} = \eta_{\mu\nu} + 2H k_\mu k_\nu, \qquad H = \frac{mr - |\psi(Y,\tau)|/2}{r^2 + a^2 \cos^2 \theta}. \tag{18}$$

The obtained solutions are time-dependent and describe the twistor-beam fluctuations of the electromagnetic vacuum and the beam-like back reaction to metric and horizon. The obtained Kerr-Schild solutions showed the existence of a classical fine-grained **Pre-Quantum geometry which takes an intermediate position between the Quantum and the usual 'smooth' Classical gravity.**

Singular pp-wave solutions (A.Peres)

Self-consistent solution of the Einstein-Maxwell equations: singular plane-fronted waves (pp-waves). Kerr-Schild form with a constant vector $k_\mu = \sqrt{2} du = dz - dt$

$g_{\mu\nu} = \eta_{\mu\nu} + 2h k_\mu k_\nu$.

Function h determines the Ricci tensor

$$R^{\mu\nu} = -k^\mu k^\nu \Box h, \tag{19}$$

where $\Box$ is a flat D'Alembertian

$$\Box = 2\partial_\zeta \partial_{\bar\zeta} + 2\partial_u \partial_v \ . \tag{20}$$

The Maxwell equations take the form $\Box \mathcal{A} = J = 0$, and can easily be integrated leading to the solutions

$$\mathcal{A}^+ = [\Phi^+(\zeta) + \Phi^-(\bar\zeta)] f^+(u) du, \tag{21}$$

$$\tag{22}$$

where $\Phi^\pm$ are arbitrary analytic functions, and function f^+ describes retarded waves.

The poles in $\Phi^+(\zeta)$ and $\Phi^-(\bar\zeta)$ lead to the appearance of singular lightlike beams (pp-waves) which propagate along the z^+ semi-axis.

pp-waves have very important quantum properties, being exact solutions in string theory with vanishing all quantum corrections (G.T. Horowitz, A.R. Steif, PRL **64** (1990) 260; A.A. Coley, PRL **89** (2002) 281601.)

Quadratic generating function F(Y) and interpretation of parameters. A.B. and G. Magli, Phys.Rev.**D 61**044017 (2000).

The considered in DKS function F is quadratic in Y,

$$F \equiv a_0 + a_1 Y + a_2 Y^2 + (qY + c)\lambda_1 - (pY + \bar{q})\lambda_2, \tag{23}$$

where the coefficients c and p are real constants and $a_0, a_1, a_2, q, \bar{q}$, are complex constants. The Killing vector of the solution is determined as

$$\hat{K} = c\partial_u + \bar{q}\partial_\zeta + q\partial_{\bar{\zeta}} - p\partial_v. \tag{24}$$

Writing the function F in the form

$$F = AY^2 + BY + C, \tag{25}$$

one can find two solutions of the equation $F = 0$ for the function $Y(x)$

$$Y_{1,2} = (-B \pm \Delta)/2A, \tag{26}$$

where $\Delta = (B^2 - 4AC)^{1/2}$.

We have also

$$\tilde{r} = -\partial F/\partial Y = -2AY - B, \tag{27}$$

and consequently

$$\tilde{r} = PZ^{-1} = \mp\Delta. \tag{28}$$

These two roots reflect the known twofoldedness of the Kerr geometry. They correspond to two different directions of congruence on positive and negative sheets of the Kerr space-time. In the stationary case

$$P = pY\bar{Y} + \bar{q}\bar{Y} + qY + c \ . \tag{29}$$

Link to the complex world line of the source. The stationary and boosted Kerr geometries are described by a straight complex world line with a real 3-velocity $\vec{v}$ in CM^4:

$$x_0^\mu(\tau) = x_0^\mu(0) + \xi^\mu \tau; \qquad \xi^\mu = (1, \vec{v}) \ . \tag{30}$$

The gauge of the complex parameter τ is chosen in such a way that $Re\ \tau$ corresponds to the real time t.

$\hat{K}$ is a Killing vector of the solution

$$\hat{K} = \partial_\tau x_0^\mu(\tau)\partial_\mu = \xi^\mu \partial_\mu \ . \tag{31}$$

$$P = \hat{K}\rho = \partial_\tau x_0^\mu(\tau) e^3_\mu \ , \tag{32}$$

where

$$\rho = \lambda_2 + \bar{Y}\lambda_1 = x^\mu e^3_\mu. \tag{33}$$

It allows one to set the relation between the parameters $p, c, q, \bar{q}$, and ξ^μ, showing that these parameters are connected with the boost of the source.

The complex initial position of the complex world line $x_0^\mu(0)$ in Eq. (30) gives six parameters for the solution, which are connected to the coefficients $a_0, \ a_1 \ a_2$. It can be decomposed as $\vec{x}_0(0) = \vec{c} + i\vec{d}$, where $\vec{c}$ and $\vec{d}$ are real 3-vectors with respect to the space

O(3)-rotation. The real part $\vec{c}$ defines the initial position of the source, and the imaginary part $\vec{d}$ defines the value and direction of the angular momentum (or the size and orientation of a singular ring).

It can be easily shown that in the rest frame, when $\vec{v} = 0, \quad \vec{d} = \vec{d}_0$, the singular ring lies in the orthogonal to $\vec{d}$ plane and has a radius $a = |\vec{d}_0|$. The corresponding angular momentum is $\vec{J} = m\vec{d}_0$.

Smooth and regular Kerr sources.

A.B., E. Elizalde, S.Hildebrandt and G. Magli, PRD (2002)

The Gürses and Gürsey ansatz $g_{\mu\nu} = \eta_{\mu\nu} + 2hk_\mu k_\nu$,

where $h = f(r)/(r^2 + a^2 \cos^2\theta)$.

Regularized solutions have tree regions:

i) the Kerr-Newman exterior, $r > r_0$, where $f(r) = mr - e^2/2$,

ii) interior $r < r_0 - \delta$, where $f(r) = f_{int}$ and function $f_{int} = \alpha r^n$, and $n \geq 4$ to suppress the singularity at $r = 0$, and provide the smoothness of the metric up to the second derivatives.

iii) a narrow intermediate region providing a smooth interpolation between i) and ii).

Non-rotating case: by $n = 4$ and $\alpha = 8\pi\Lambda/6$,

interior is a space-time of constant curvature $R = -24\alpha$.

Energy density of source $\rho = \frac{1}{4\pi}(f'r - f)/\Sigma^2$,

tangential and radial pressures $p_{rad} = -\rho, \quad p_{tan} = \rho - \frac{1}{8\pi} f''/\Sigma$, where $\Sigma = r^2$.

There is a de Sitter interior for $\alpha > 0$, and anti de Sitter interior for $\alpha < 0$. Interior is flat if $\alpha = 0$.

The resulting sources may be considered as the bags filled by a special matter with positive ($\alpha > 0$) or negative ($\alpha < 0$) energy density. The transfer from the external electro-vacuum solution to the internal region (source) may be considered as a phase transition from 'true' to 'false' vacuum. Assuming that transition region iii) is very thin, one can consider the following useful graphical representation.

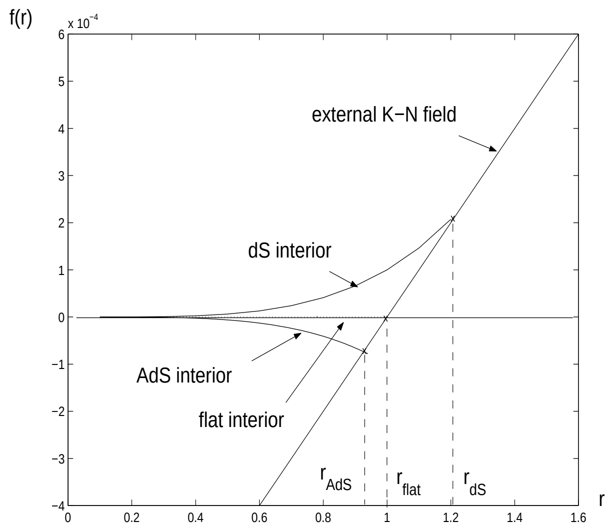

Figure 6: Regularization of the Kerr spinning particle by matching the external field with dS, flat or AdS interior.

The point of phase transition r_0 is determined by the equation $f_{int}(r_0) = f_{KN}(r_0)$ which yields

$$m = \frac{e^2}{2r_0} + \frac{4}{3}\pi r_0^3 \rho. \tag{34}$$

The first term on the right side is electromagnetic mass of a charged sphere with radius r_0, $M_{em}(r_0) = \frac{e^2}{2r_0}$, while the second term is the mass of this sphere filled by a material with a homogenous density ρ, $M_m = \frac{4}{3}\pi r_0^3 \rho$. Thus, the point of intersection r_0 acquires a deep physical meaning, providing the energy balance by the mass formation.

Transfer to rotating case. One has to set $\Sigma = r^2 + a^2 \cos^2\theta$, and consider r and θ as the oblate spheroidal coordinates.

The Kerr source represents a disk with the boundary $r = r_0$ which rotates rigidly. In the corotating with disk coordinate system, the matter of the disk looks homogenous distributed, however, because of the relativistic effects the energy-momentum tensor increases strongly near the boundary of the disk.

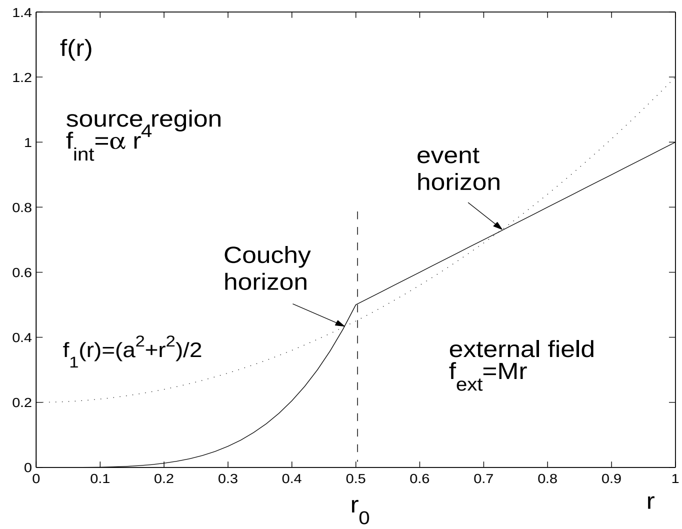

Figure 7: Matching the (rotating) internal "de Sitter" source with the external Kerr-Schild field. The dotted line $f_1(r) = (r^2 + a^2)/2$ determines graphically the position of horizons as the roots of the equation $f(r) = f_1(r)$.

In the limit of a very thin disk a stringy singularity develops on the border of disk. This case corresponds to the Israel-Hamity source 1970-1976.

The Kerr-Newman spinning particle with $J = \frac{1}{2}\hbar$, acquires the form of a relativistically rotating disk which has the form of a highly oblate ellipsoid with the thickness $r_0 \sim r_e$ and the Compton radius $a = \frac{1}{2}\hbar/m$. Interior of the disk represents a "false"

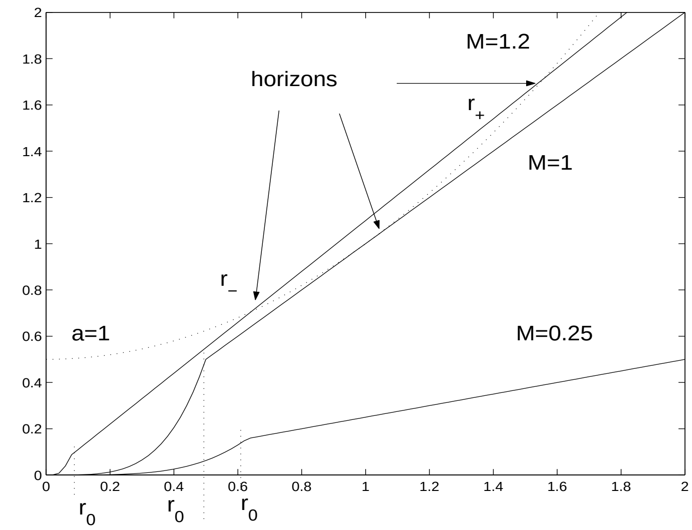

Figure 8: The sources with different masses M and matter densities ρ. Sources form the rotating disks with radius $\sim a$ and thickness $\sim r_0$ which depends on the matter density $r_0 = (\frac{3M}{4\pi\rho})^{1/3}$. The formation of the black hole horizons is shown for $a^2 < M^2$.

vacuum having superconducting properties which are modelled by the Higgs field.

Properties of the disklike Kerr source

- the disk is oblate and rigidly rotating,
- the rotation is relativistic, so the board of the disk is moving with the speed which is close to the speed of light,
- the stress-energy tensor of the matter of the disk has an exotic form resembling a special condensed vacuum state (de Sitter, flat or anti de Sitter vacua).
- electromagnetic properties of the matter of the disk are close to superconductor,
- the charge, strong magnetic and gravitational fields are concentrated on the stringy board of the disk, and are partially compensated from the oppositely charged part of the disk surface. It yields a very specific form of the electromagnetic field (see fig.4, and fig.5).
- finally, the main property of Kerr-Schild source - the relation $J = Ma$ between the angular momentum J, mass M, and the radius of the Kerr ring a.

Complex Kerr source, complex shift. Appel 1887!

A point-like charge e, placed on the complex z-axis $(x_0, y_0, z_0) = (0, 0, ia)$, gives the real Appel potential

$$\phi_a = Re\ e/\tilde{r}, \tag{35}$$

where $\tilde{r} = r + ia\cos\theta$ is the Kerr complex radial coordinate and r and θ are the oblate spheroidal coordinates. In the Cartesian coordinates x, y, z, t

$$\tilde{r} = [(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2]^{1/2} = [x^2 + y^2 + (z - ia)^2]^{1/2}. \tag{36}$$

Singularity of the Appel potential ϕ_a corresponds to $r = \cos\theta = 0$, and therefore, the singular ring $z = 0, \quad x^2 + y^2 = a^2$ is a branch line of space-time for two sheets, just similar to the Kerr singular ring.

Appel potential describes *exactly* the em field of the Kerr-Newman solution on the auxiliary M^4.

Complex world line and complex Kerr string.

If the Appel source is shifted to a complex point of space $(x_o, y_o, z_o) - (0, 0, ia)$, it can be considered as a mysterious "particle" propagating along a *complex world-line* $x_0^\mu(\tau)$ in CM^4 and parametrized by a complex time τ. The complex source of the Kerr-Newman solution has just the same origin and can be described by means of a complex retarded-time construction for the Kerr geometry.

The objects described by the complex world-lines occupy an intermediate position between particle and string. Like a string they form two-dimensional surfaces or world-sheets in space-time. In many respects this source is similar to the "mysterious" $N = 2$ complex string of superstring theory.

The Kerr congruence may be understood as a track of the null planes of the family of complex light cones emanating from the points of the complex world line $x_0^\mu(\tau)$ in the retarded-time construction.

Complex retarded-time parameter.

Parameter τ may be defined for each point x of the Kerr space-time and plays the role of a complex retarded-time parameter. Its value for a given point x may be defined by L-projection, using the solution $Y(x)$ and forming the twistor parameters $\lambda_1, \quad \lambda_2$ which fix a left null plane. The points x^μ and x_0^μ are connected by the left null plane spanned by the null vectors e^1 and e^3.

The point of intersection of this plane with the complex world-line $x_0(\tau)$ gives the value of the "left" retarded time τ_L, which is in fact a complex scalar function on the (complex) space-time $\tau_L(x)$.

By using the null plane equation, one can get a retarded-advanced time equation

$$\tau = t \mp \tilde{r} + \vec{v}\vec{R}. \tag{37}$$

For the stationary Kerr solution $\tilde{r} = r + ia\cos\theta$, and one can see that the second root $Y_2(x)$ corresponds to a transfer to the negative sheet of the metric: $r \rightarrow -r; \quad \vec{R} \rightarrow -\vec{R}$, with a simultaneous complex conjugation $ia \rightarrow -ia$.

The analytical twistorial structure of the Kerr spinning particle leads to the appearance of an extra axial stringy system. As a result, the Kerr spinning particle acquires a simple stringy skeleton which is formed by a topological coupling of the Kerr circular string and the axial stringy system. The projective spinor coordinate Y is a projection of sphere on complex plane. It is singular at $\theta = \pi$, and such a singularity will be present in any holomorphic function $\psi(Y)$. Therefore, *all the aligned e.m. solutions turn out to be singular at some angular direction* θ. The simplest modes

$$\psi_n = qY^n \exp i\omega_n\tau \equiv q(\tan\frac{\theta}{2})^n \exp i(n\phi + \omega_n\tau) \qquad (38)$$

can be numbered by index $n = \pm1, \pm2, ...$

The leading wave terms are

$$\mathcal{F}|_{wave} = f_R \, d\zeta \wedge du + f_L \, d\bar\zeta \wedge dv, \qquad (39)$$

where

$$f_R = (AZ)_{,1}\,; \qquad f_L = 2Y\psi(Z/P)^2 + Y^2(AZ)_{,1} \qquad (40)$$

are the factors describing the "left" and "right" waves propagating along the z^- and z^+ semi-axis correspondingly.

The parameter $\tau = t - r - ia\cos\theta$ takes near the z-axis the values $\tau_+ = \tau|_{z^+} = t - z - ia, \quad \tau_- = \tau|_{z^-} = t + z + ia.$

The leading wave for $n = 1$,
$\mathcal{F}_1^- = \frac{4qe^{i2\phi + i\omega_1\tau_-}}{\rho^2} \, d\bar\zeta \wedge dv,$
propagates to $z = -\infty$ along the z^- semi-axis.

The leading wave for $n = -1$,
$\mathcal{F}^+_{-1} = -\frac{4qe^{-i2\phi+i\omega_{-1}\tau_+}}{\rho^2}\, d\zeta \wedge du$,
is singular at z^+ semi-axis and propagates to $z = +\infty$.

The $n = \pm 1$ partial solutions represent asymptotically the singular plane-fronted e.m. waves propagating without damping.

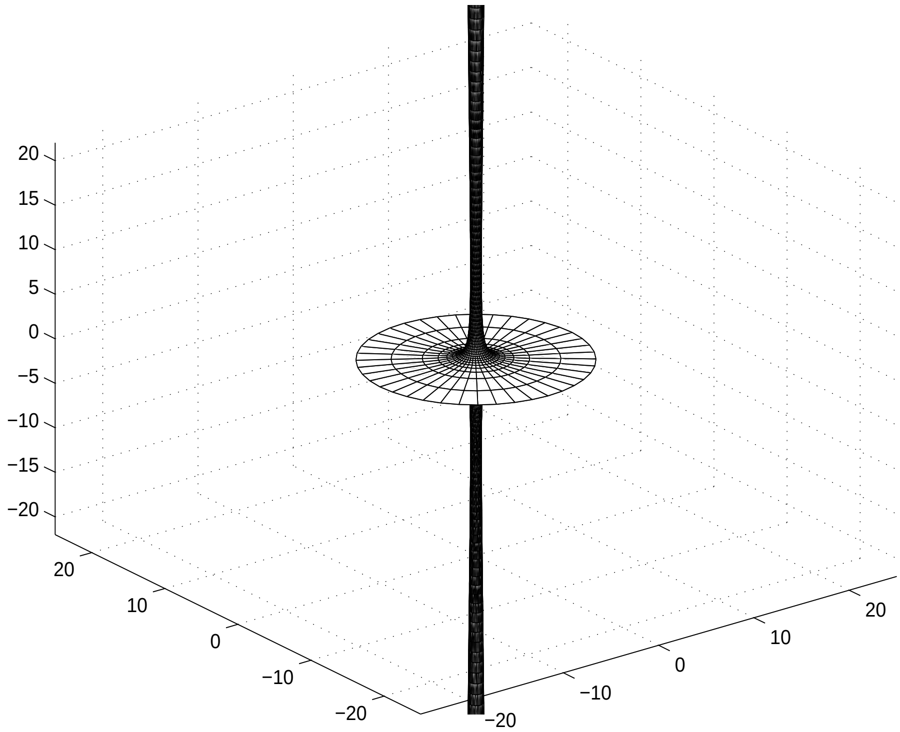

Figure 9: The Kerr disk-like source and two axial semi-infinite beams.