Superstring Field Theory with the Pure Spinor Formalism

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Ref: NB, hep-th/0509120 "Pure spinor formalism as an $N=2$ top. string"
Yuri Aisaka and NB, hep-th/0903.3443 "Pure spinor vertex op's in Siegel gauge and loop amp. regularization"

MOTIVATION

Most approaches to superstring field theory are based on RNS formalism.

- Good for describing NS sector (both GSO(+) and GSO(−)), but awkward for describing R sector (e.g. spin fields, bosonization, pictures, etc..)
- Can describe NS sector with manifest $d=10$ covariance, but R sector appears to require breaking of manifest Lorentz covariance,
- Spacetime susy is not manifest.

Can define light-cone ssft using GS formalism

- SO(8) susy is manifest. Requires contact terms.

To define covariant ssft, use pure spinor formalism

- ssft equations of motion are manifestly SO(9,1) super-Poincaré invariant.
- Can formally construct a cubic open ssft action.
- Gauge-fixing is problematic.

Pure spinor formalism is based on superspace description of <u>super YM</u>

$N=1$ $d=10$ superspace: x^m, θ^α $\quad m=0$ to 9, $\alpha=1$ to 16

$$D_\alpha = \frac{\partial}{\partial\theta^\alpha} + \frac{1}{2}\gamma^m_{\alpha\beta}\theta^\beta \frac{\partial}{\partial x^m}, \quad \{D_\alpha, D_\beta\} = \gamma^m_{\alpha\beta}\partial_m$$

To describe super-YM, covariantize superspace derivatives

$$D_\alpha \to \nabla_\alpha \equiv D_\alpha + A_\alpha(x,\theta), \quad \partial_m \to \nabla_m \equiv \partial_m + A_m(x,\theta)$$

Superfields $A_\alpha(x,\theta)$, $A_m(x,\theta)$ are defined up to gauge transformation

$$\delta A_\alpha(x,\theta) = \nabla_\alpha \Omega(x,\theta), \quad \delta A_m(x,\theta) = \nabla_m \Omega(x,\theta)$$

Super-YM eqns. of motion are implied by $\{\nabla_\alpha, \nabla_\beta\} = \gamma^m_{\alpha\beta}\nabla_m$

$$\Rightarrow \quad \gamma_m^{\alpha\beta}\left(D_\alpha A_\beta + D_\beta A_\alpha + \{A_\alpha, A_\beta\}\right) = 16 A_m$$

$$\gamma_{mnpqr}^{\alpha\beta}\left(D_\alpha A_\beta + D_\beta A_\alpha + \{A_\alpha, A_\beta\}\right) = 0$$

$$\Rightarrow \quad A_\alpha = \frac{1}{2}a_m(x)(\gamma^m\theta)_\alpha + \chi^\beta(x)(\gamma^m\theta)_\alpha(\gamma_m\theta)_\beta + \ldots, \quad A_m = a_m(x) + \ldots$$

Gauge transf. $\delta A_\alpha = \nabla_\alpha \Omega$ and eq. of motion $\gamma^{\alpha\beta}_{mnpqr} \{ \nabla_\alpha, \nabla_\beta \} = 0$ can be described using the nilpotent operator

$$\tilde{Q} = \lambda^\alpha \nabla_\alpha \equiv \lambda^\alpha (D_\alpha + A_\alpha(x,\theta))$$ Howe '91

where λ^α is a "pure spinor" satisfying $\boxed{\lambda^\alpha \gamma^m_{\alpha\beta} \lambda^\beta = 0}$ for $m=0$ to 9.

To relate to open ssft, define linearized vertex operator

$V = \lambda^\alpha A_\alpha(x,\theta)$ for massless super-YM state.

$(\tilde{Q})^2 = 0 \Rightarrow QV + \frac{1}{2}\{V, V\} = 0$ where $Q = \lambda^\alpha D_\alpha$.

Easy to generalize to massive states by allowing V to depend on non-zero modes of worldsheet variables.

To construct a cubic action whose eqn. of motion is $QV + V * V = 0$, need to introduce midpoint operator.

Gauge-fixing is problematic.

Pure spinor worldsheet action:

$$S = \frac{1}{\alpha'}\int d^2z \left[\frac{1}{2}\partial x^m \bar\partial x_m + p_\alpha \bar\partial \theta^\alpha + w_\alpha \bar\partial \lambda^\alpha + \bar p_\alpha \partial \bar\theta^\alpha + \bar w_\alpha \partial \bar\lambda^\alpha\right]$$

$$\boxed{\lambda\gamma^m\lambda = \bar\lambda\gamma^m\bar\lambda = 0}$$

D_9-brane bdy. cond's: $\partial_\sigma x^m = 0$, $\theta^\alpha = \bar\theta^\alpha$, $\lambda^\alpha = \bar\lambda^\alpha$, $p_\alpha = \bar p_\alpha$, $w_\alpha = \bar w_\alpha$,

Action is in conformal gauge with BRST operator defined as

$Q = \int dz\, \lambda^\alpha d_\alpha$ where $d_\alpha \equiv p_\alpha - \frac{1}{2}(\gamma^m\theta)_\alpha \partial x_m - \frac{1}{8}(\gamma^m\theta)_\alpha(\theta\gamma_m\partial\theta)$ is spacetime supersymmetric.

$$d_\alpha(y)\, d_\beta(z) \to -\frac{\gamma^m_{\alpha\beta}\Pi_m}{y-z} \quad \text{where } \Pi^m \equiv \partial x^m + \frac{1}{2}\theta\gamma^m\partial\theta \;\Rightarrow\; Q^2 = 0$$

Stress tensor $T = \frac{1}{2}\partial x^m \partial x_m + p_\alpha \partial\theta^\alpha + w_\alpha \partial\lambda^\alpha$ has no conformal anomaly since $\lambda\gamma^m\lambda = 0 \Rightarrow \lambda^\alpha$ has 11 indep. components.

Physical states are defined as ghost-number +1 states in cohomology of Q.

Ex: Massless sYM: $V = \lambda^\alpha A_\alpha(x,\theta)$

Massive superspin 2: $V = \lambda^\alpha(\partial\theta^\beta A_{\alpha\beta} + \Pi^m B_{\alpha m} + d_\beta C_\alpha^\beta + \lambda^\gamma w_\gamma D_{\alpha\gamma}^\delta) + \partial\lambda^\alpha E_\alpha$

Using "minimal" pure spinor formalism, there is no composite field b which satisfies $\{Q, b\} = T$.

Scattering amplitudes:

3-point tree amplitude: $\mathcal{A} = \langle V V V \rangle$ using measure factor

$$\langle (\lambda\gamma^m\theta)(\lambda\gamma^n\theta)(\lambda\gamma^p\theta)(\theta\gamma_{mnp}\theta)\rangle \equiv \langle \lambda^3\theta^5\rangle \equiv 1.$$

$\lambda^3\theta^5$ is unique state of +3 ghost-number in cohom. of Q (like $c\partial c\partial^2 c$).

States of +2 ghost-number in cohom. of Q are "antifields" (like $c\partial c V$)

Antifield for super-YM is $V^* = \lambda^\alpha\lambda^\beta A_{\alpha\beta}(x,\theta) = \ldots + \chi^*_\gamma(x)(\lambda^2\theta^3)^\gamma + a^*_m(x)(\lambda^2\theta^4)^m$

Naively can define open ssft action

$$S = \frac{1}{g^2}\langle \tfrac{1}{2} VQV + \tfrac{1}{3} V*V*V\rangle \qquad \text{(Schwarz + Witten, '01 unpublished)}$$

But $\langle\lambda^3\theta^5\rangle \equiv 1$ is only positive-definite norm if vertex operators are BRST-closed (for example, states with more than 5 θ's decouple).

To construct a consistent measure factor for off-shell states, need to use "non-minimal pure spinor" formalism.

Non-minimal pure spinor formalism:

Add non-minimal "quartet" $(\underbrace{\tilde{\lambda}_\alpha, \tilde{\omega}^\alpha}_{\text{bosons}}; \underbrace{r_\alpha, s^\alpha}_{\text{fermions}})$ s.t. $\tilde{\lambda}\gamma^m\tilde{\lambda} = r\gamma^m\tilde{\lambda} = 0$

$$S_{\text{non-min}} = S_{\text{min}} + \frac{1}{\alpha'}\int d^2z \left[\tilde{\omega}^\alpha \bar{\partial}\tilde{\lambda}_\alpha + s^\alpha \bar{\partial} r_\alpha + \tilde{\bar{\omega}}^\alpha \partial \tilde{\bar{\lambda}}_\alpha + \bar{s}^\alpha \partial \bar{r}_\alpha \right]$$

$$Q_{\text{non-min}} = Q_{\text{min}} + \int dz\, \tilde{\omega}^\alpha r_\alpha \quad \Rightarrow \text{non-minimal variables do not affect cohomology}$$

Measure factor $\langle \lambda^3 \theta^5 \rangle = 1$ can be obtained from functional integration if non-minimal variables are used to "regularize" 0/0 ambiguities.

Define BRST-inv. regulator $\mathcal{N}_\rho = e^{\rho\{Q,\chi\}} = e^{-\rho(\lambda^\alpha\tilde{\lambda}_\alpha + \theta^\alpha r_\alpha)}$

$\mathcal{N} = 1 + \{Q, \Omega\} \Rightarrow$ on-shell amplitudes are indep. of constant ρ and location of $\mathcal{N}_\rho$

$$\langle f(x,\theta,\lambda,\tilde{\lambda},r)\rangle_{\text{non-min}} \equiv \int d^{10}x \int d^{16}\theta \int d^{11}\lambda \int d^{11}\tilde{\lambda} \int d^{11}r\; \mathcal{N}_\rho\, f(x,\theta,\lambda,\tilde{\lambda},r)$$

$\langle \lambda^3\theta^5 \rangle_{\text{non-min}} = 1 \Rightarrow$ can define $\boxed{S_\rho = \frac{1}{g^2}\left\langle \frac{1}{2} V Q_{\text{non-min}} V + \frac{1}{3} V * V * V \right\rangle_{\text{non-min}}}$

where string field V can depend on minimal and non-minimal variables.

$\mathcal{N}_\rho$ is inserted at midpoint interaction (like Y^2 in RNS cubic action).

$\eta_\rho = e^{-\rho(\lambda^\alpha \tilde{\lambda}_\alpha + \theta^\alpha r_\alpha)} \;\ni\; (\eta_\rho)^{-1} = e^{\rho(\lambda^\alpha \tilde{\lambda}_\alpha + \theta^\alpha r_\alpha)}$ is inverse of η_ρ.

So eqn. of motion $\eta_\rho (Q_{non\text{-}min} V + V * V) = 0 \Rightarrow Q_{non\text{-}min} V + V * V = 0$.

Similar construction may be possible using non-min. version of RNS (NB + W. Siegel, '09; M. Kroyter, work in progress)

S_ρ is not manifestly spacetime supersymmetric because η_ρ depends on θ^α.

Massless sector of string field V requires infinite number of fields.

$$V = \lambda^\alpha A^{(0)}_\alpha + \lambda^\alpha \lambda^\beta \tilde{\lambda}_\gamma A^{\gamma\,(1)}_{\alpha\beta} + \lambda^\alpha \lambda^\beta \lambda^\gamma \tilde{\lambda}_\delta \tilde{\lambda}_\kappa A^{\delta\kappa\,(2)}_{\alpha\beta\gamma} + \ldots = \sum_{n=0}^{\infty} V^{(n)}$$

where $V^{(n)}$ has "non-minimal charge" $+n$.

When expressed in terms of $V^{(n)}$,

$$S_\rho = \frac{1}{g^2}\left\langle \frac{1}{2} V Q_{non\text{-}min} V + \frac{1}{3} V * V * V \right\rangle = \sum_{n=0}^{\infty} \rho^{-n} S^{(n)}_{\rho=1}$$

where $S^{(n)}_{\rho=1} = \frac{1}{g^2}\left\langle \frac{1}{2}\sum_{p=0}^{n} V^{(p)} Q_{non\text{-}min} V^{(n-p)} + \frac{1}{3}\sum_{p=0}^{n}\sum_{q=0}^{n-p} V^{(p)} * V^{(q)} * V^{(n-p-q)} \right\rangle$

Need for infinite number of fields can be seen from spin ½ kinetic term $\int d^{10}x\, (\chi \not{\partial} \chi)$ which requires non-minimal variables to be expressed as $\langle \Phi Q \Phi \rangle$ action (NB, M. Hatsuda, W. Siegel, '91)

Siegel gauge:

To quantize $\mathcal{S}_\rho = \frac{1}{g^2}\langle \frac{1}{2} V Q_{non\text{-}min} V + \frac{1}{3} V * V * V \rangle_{non\text{-}min}$, need to gauge-fix string field V.

Natural gauge-fixing is $b_0 V = 0$ where $\{Q_{non\text{-}min}, b_0\} = L_0$.

Using non-minimal variables, can construct composite operator b s.t. $\{Q, b\} = T$

$$b = s^\alpha \partial \tilde{\lambda}_\alpha + \frac{\tilde{\lambda}_\alpha (\Pi d + \lambda w \partial \theta)^\alpha}{\lambda^\beta \tilde{\lambda}_\beta} + \frac{\tilde{\lambda}_\alpha r_\gamma (d d + \lambda w \Pi)^{\alpha\gamma}}{(\lambda^\beta \tilde{\lambda}_\beta)^2} + \frac{\tilde{\lambda}_\alpha r_\gamma r_\delta (d \lambda w)^{\alpha\gamma\delta}}{(\lambda^\beta \tilde{\lambda}_\beta)^3} + \frac{\tilde{\lambda}_\alpha r_\gamma r_\delta r_\rho (\lambda\lambda w w)^{\alpha\gamma\delta\rho}}{(\lambda^\beta \tilde{\lambda}_\beta)^4}$$

To construct vertex operator in Siegel gauge, start with vertex op. for <u>antifield</u> of ghost-number $+2$ V^*, and define $V_{siegel} = b_0 V^*$ (Yuri Aisaka + NB, 0903.3443).

For super-YM states, $V^* = \lambda^\alpha \lambda^\beta A^*_{\alpha\beta}(x,\theta) \Rightarrow V_{siegel} = \sum_{n=1}^{4} \frac{\lambda^\alpha \lambda^\beta \tilde{\lambda}_\gamma A^{\gamma(n)}_{\alpha\beta}(x,\theta,r)}{(\lambda^\delta \tilde{\lambda}_\delta)^n}$

So vertex operators in Siegel gauge <u>cannot</u> be expressed as a polynomial

$$V = \lambda^\alpha A^{(0)}_\alpha + \lambda^\alpha \lambda^\beta \tilde{\lambda}_\gamma A^{\gamma(1)}_{\alpha\beta} + \dots$$

Unclear what is the correct defn. of "Hilbert space" for string field V.

CONCLUSIONS

- Open superstring field theory eqns. of motion can be expressed in manifestly d=10 super-Poincaré invariant manner as $QV + V*V = 0$ using "minimal pure spinor" formalism.

- To construct action, need to introduce non-minimal variables and insert BRST-invariant regulator $\mathcal{N}_\rho$ at midpoint.

- Unlike Y^2 insertion in RNS cubic action, $\mathcal{N}_\rho$ can be continuously deformed to identity operator, and is invertible.

- Resulting cubic action S_ρ depends on parameter ρ and involves an infinite number of fields already in the massless sector.

- Gauge-fixing to Siegel gauge requires that Hilbert space of string field includes $\frac{1}{(\lambda\bar{\lambda})}$ dependence.

- Consistent defn. of off-shell Hilbert space is not yet understood.