BSFT and Closed Strings

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MB, I. Sachs, S. Shatashvili, 2004
MB, I. Sachs, 2006
MB, I. Sachs, 2008
MB, 2008

Open strings vs. closed strings

- **S-matrix:**
  closed string poles in open string scattering amplitudes
  unitarity in OSFT?
- **2d string theory (Liouville):** explicit duality
- **tachyon condensation:** open string completeness conjecture
- **topological string theory:** e.g. Hochschild complex

Open strings vs. closed strings

- **BCFT:** open and closed string moduli
- open moduli space changes significantly under closed string deformations
- dramatic effects on D-branes

  [MB, Brunner, Gaberdiel, 2007]

  [MB, Wood, 2008]

- how do closed string deformations appear in OSFT?
- background (in)dependence?

Outline

Outline

BSFT in a nutshell

- $S = \int_\Sigma L(G_{\mu\nu}; B_{\mu\nu}; X^\mu) + \int_{\partial\Sigma} b(t^I; X^\mu)$
  - L defines a CFT as 'background' with massless fields as moduli
  - b is a boundary term with a 'complete' set of couplings t^I
  - and vertex operators $V_I = \frac{\partial b(t^J)}{\partial t^I}\Big|_{t^J=0}$
  - b breaks conformal invariance (on the boundary only)
- classical solutions of SFT: ghost + matter CFT
- idea: consider the space of all *boundary deformations* $\mathscr{O}$ *(ghost number 1)*

  $S = S_{\text{matter}} + S_{\text{ghosts}} + \oint b_{-1}\mathscr{O}$

  $b_{-1} = \oint_{C\to\partial D} b, \quad Q_B = \oint_{C\to\partial D} J_B$

BSFT in a nutshell

- construct action via BV:
  use master equation (choose anti-bracket) + classical solutions
- SFT action *(form on coupling space)*:

$$d\mathscr{S} = \langle \oint d\mathscr{O} \left\{ Q_B, \oint \mathscr{O} \right\} \rangle$$

[Witten]

- matter/ghost decoupling: $\mathscr{O} = c\mathscr{V} = ct^I \mathscr{V}_I$

$$\mathscr{S} = \left(1 - \beta^I(t) \frac{\partial}{\partial t^I} \right) \mathscr{Z}(t)$$

[Shatashvili]

- generating functional $\mathscr{Z}(t^I) = \int DX \, e^{-S[X]}$

Space of boundary couplings

- consider flat background $\mathbb{R}^{1,25}$ with string field X
  derivative expansion:
  $\mathscr{V}(X) = T(X) + A_\mu(X)\dot{X}^\mu + \textit{massive modes}$
- $\mathscr{S}$ is a functional of *infinitely many* modes *(renormalizability!)*
- these might be expansions of non-local interactions *[Li, Witten]*

  $\oint\oint X(\sigma)u(\sigma,\sigma')X(\sigma')$ can be expanded in derivatives $\sum_n \oint X t_n \partial_\sigma^{(n)} X$ with $u^n = \sum_m (in)^m t_m$
- such 'collectively excited' couplings appear naturally and get interpretation from closed string sector

Background dependence?

$$\mathscr{M} = (G_{\mu\nu}, B_{\mu\nu}) \qquad \longleftrightarrow \qquad \mathscr{M}' = (G'_{\mu\nu}, B'_{\mu\nu})$$

- OS spectrum may change
- OS β-equations may change
- OS conformal point shifted $t_* \to t'_*$
- BSFT action $\mathscr{S} = (1 - \beta^I \partial_I)\mathscr{Z}$ defined pointwise

NB: reminiscent of the situation in open-closed moduli spaces,
Picard-Fuchs equation systems

Background independence!

$$\mathscr{M} = (G_{\mu\nu}, B_{\mu\nu}) \qquad \longleftrightarrow \qquad \mathscr{M}' = (G'_{\mu\nu}, B'_{\mu\nu})$$

$$\mathscr{S}_{\mathscr{M}}(t^I) \qquad \longleftrightarrow \qquad \mathscr{S}_{\mathscr{M}'}(t^I + \Delta t^I; \tau^I)$$

- new open string background $t'_* = t_* + \Delta t$
- the original couplings t^I still describe the open strings
- surprise: no need to integrate out closed strings, due to a factorization property; no higher order α' terms in the boundary interaction term b

$$\mathscr{Z}_{\mathscr{M}} = \mathscr{Z}_0 \, \mathscr{Z}_{\mathscr{M}'}$$

- but: generically non-local couplings τ^I appear

Outline

Re-formulation of BSFT

splitting of fields

- how to parametrize space of boundary-deformed 2d CFTs?
- {space of boundary deformations} $\longleftrightarrow$ {space of functions f on S^1}

  (also parametrizes all boundary conditions, after integration)
- unique decomposition: $X = X_0 + X_b$
  where $X_0| = x_0$ (D0) and $\Delta X_b = 0$ (harmonic)
- thus $X_b = X_b[f]$ with $X_b(\tau, \sigma)| = f(\sigma)$
- $S(X) = \int d^2z\, \partial X \bar{\partial} X = S(X_0) + S(X_b) + W(X_0, X_b)$
- with $S(X_b) = \oint d\sigma d\sigma'\ f(\sigma) H(\sigma - \sigma') f(\sigma')$
- and $H(\sigma) = \sum_n \oint d\sigma |n| e^{in\sigma}$

Re-formulation of BSFT

more on the boundary action

- boundary data $f(\sigma)$ can be decomposed by holomorphic/anti-holomorphic extendibility on the disk
- $f(\sigma) = f_+(\sigma) + f_-(\sigma)$
$$\longrightarrow f_+(z) + f_-(\bar{z}) = f(z, \bar{z}) = X_b(z, \bar{z})$$
- action of H: $\quad Hf = \partial_\sigma f_+ - \partial_\sigma f_- = \partial_n X_b$
- boundary action: $\oint f_+ df_- = \oint X_b \partial_n X_b$
observation: 2-cocycle

Re-formulation of BSFT

partition function

- replace $\mathscr{Z}[t]$ by $\mathscr{Z}[f]$
  $\mathscr{Z}[f]$ – partition function with fixed boundary conditions f:
  $\mathscr{Z}[t] = \int Df\ e^{-I[t]} \mathscr{Z}[f]$
- path-integral factorizes:

$$\mathscr{Z}[f] = \int_{\text{D0 b.c.}} DX_0\ e^{-S[X_0]} \times e^{-\tilde{S}[f]} = Z_0\ e^{-\tilde{S}[f]}$$

- no 'interaction' between 'bulk part' and 'boundary part' of X
- X_0 has not been 'integrated out'! I.e. arbitrary boundary interactions allowed

Re-formulation of BSFT

partition function

- factorization allows to re-define BSFT-action
  (requires choice of measure for f-path-integral)

$$\mathscr{S} \longrightarrow \mathscr{S}' = \mathscr{S} \,/\, \mathscr{Z}_0$$

- everything formulated in terms of ‘boundary fields’ f
- $\mathscr{S}' = \mathscr{S} \,/\, \mathscr{Z}_0$ is well-defined
- change in closed string background can be studied in BSFT
- how generic is this factorization property?

Simple examples

Starting from the free boson $\mathscr{M} = (G_{\mu\nu} = \eta_{\mu\nu}, B_{\mu\nu} = 0, R)$

- a shift in the Kalb-Ramond field yields

$$\delta B_{\mu\nu} \oint X_b^\mu dX_b^\nu \qquad \longrightarrow \text{gauge field}$$

- a shift in the compactification radius R yields

$$\frac{\delta R}{R} \oint X_b \partial_n X_b$$

Outline

Remarks on WZW, closed string only

- model for non-trivial (flat) closed string background
- $S_{WZW}(g) = \kappa L(g) + \kappa \Gamma(g)$

$$L = \mathrm{Tr} \int g^{-1} \partial g g^{-1} \bar{\partial} g$$
$$\Gamma = \mathrm{Tr} \int w_3$$
$$w_3 = (dgg^{-1})^{\wedge 3}$$

- for w_3 extension of g to interior of closed worldsheet necessary
- no ambiguities in partition function

Remarks on boundary WZW

- for simplicity assume $H^3 = 0$
- w_3 can be pulled to the boundary, $w_3 = dw_2$
- ambiguity: w_2 defined up to closed form $d\beta$
- thus for the action one needs to specify a one-form β:

$$\Gamma^\beta = \Gamma + \oint \beta$$

- β lives on the boundary only
- some assumptions needed to fix β (below)

Factorization for boundary WZW

- Splitting of the fields:

$$g = g_0\, k \qquad \text{in analogy to } X = X_0 + X_b$$
$$k = h(z)\bar{h}(\bar{z}) \qquad \text{in analogy to } X_b = f_+(z) + f_-(\bar{z})$$

- g_0 satisfies D0-boundary conditions, $g_0| = \text{const}$
  k satisfies classical EOM, $\partial k k^{-1} = 0 = k^{-1}\bar{\partial} k$

- $S(g_0 k)$ determined by Polyakov-Wiegmann formula, but needs extension for boundary WZW

Boundary Polyakov-Wiegmann

- only Γ may be problematic.
  for closed string: $w_3(g_1 g_2) = w_3(g_1) + w_3(g_2) + G(g_1, g_2)$
- for open string one can show, that
  $w_3(g_1 g_2) - w_3(g_1) - w_3(g_2) - G(g_1, g_2)$ is a 2-cocycle on LG
- and its integral over the worldsheet depends only on its values on the boundary and is independent of the choice of extension
- for $g = g_0\, k$ this integral vanishes
- finally,

$$S_{WZW}(g) = S_{WZW}(g_0) + L(k) + \Gamma^{\beta}(k) + W(g_0, k)$$

with $W(g_0, k) = \mathrm{Tr} \int g_0^{-1} \bar{\partial} g_0 \partial k k^{-1}$

Factorization

$$S_{WZW}(g) = S_{WZW}(g_0) + L(k) + \Gamma^{\beta}(k) + W(g_0, k)$$

- a priori no decoupling of g_0 and k, due to presence of W
- but still, the partition function factorizes *(main technical result)*:

$$\left\langle \left(\mathrm{Tr} \int g_0^{-1} \bar{\partial} g_0 \partial k k^{-1} \right)^n \right\rangle_{g_0} = 0$$

- thus W does not contribute
- $Z[k] = Z_0 e^{-\tilde{S}[k]}$ is still true

Outline

The boundary action

$$S(k) = L(k) + \Gamma^{\beta}(k)$$

- explicit expression wanted
- using the Ansatz $k(z,\bar{z}) = h(z)\bar{h}(\bar{z})$ and boundary Polyakov-Wiegmann, we get

$$\Gamma^{\beta}(k) = -L(k) + \int \alpha(h,\bar{h})$$

  where α is a 2-cocycle
- therefore factorization gives

$$Z[k] = Z_0\, e^{-\int \alpha(h,\bar{h}) - \oint \beta}$$

- we cannot calculate the boundary action, but we can make a guess based on some basic properties

The boundary action

the chosen action must ...

- ... lead to the same algebraic boundary conditions as the 'full' WZW model
- ... obey the cocycle conditions
- ... give the correct large-radius limit
- our Ansatz:

$$\tilde{S} = L(k) = \int k^{-1}\partial k k^{-1}\bar{\partial} k = \int h^{-1}\partial h \bar{\partial}\bar{h}\bar{h}^{-1}$$

(can be written as a surface integral)

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... SL(2), SU(2), Nappi-Witten ...

Outline

Curved background, an example

WZW on SU(2)

- symmetries
- maximally symmetric D-branes known $\leftrightarrow$ conjugacy classes, discrete
- parametrization:

$$k = e^{i\lambda X^{\mu} T_{\mu}}$$

λ – $\kappa^{-\frac{1}{2}}$, expansion parameter
T – generators of SU(2)
large radius limit: $\lambda \to 0 \quad \Leftrightarrow \quad \kappa \to \infty$

- in the same way:

$$h(z) = e^{i\lambda f_{+}^{\mu} T_{\mu}} \qquad \bar{h}(\bar{z}) = e^{i\lambda f_{-}^{\mu} T_{\mu}}$$

Coordinates

Relation between flat and curved coordinates

$$X^\mu = f_+^\mu + f_-^\mu - \lambda \varepsilon_{\mu\nu\kappa} f_+^\nu f_-^\kappa + \mathcal{O}(\lambda^2)$$

- continuation: $f_+ \to f(z) \qquad f_- \to \bar{f}(\bar{z})$
- technical issue: enforces $SU(2)^{\mathbb{C}}$

Non-local deformations

- definition of a path-integral:

$$\int \left[\delta k k^{-1}\right] e^{-\frac{1}{(i\lambda)^2}\int \partial k k^{-1}\bar{\partial} k k^{-1}}$$

- express everything in $f, \bar{f}$
- action:

$$S = s\sum_{m=1}^{\infty} m f_m \bar{f}_m - \frac{\lambda}{2}\left\{V_\alpha - \bar{V}_\alpha + \lambda V_\beta + \lambda V_\gamma + \lambda \bar{V}_\gamma\right\}$$

$$V_\alpha = \sum (c-b)\delta_{a,c+b}\varepsilon_{\mu\nu\lambda} f_c^\mu f_b^\nu \bar{f}_a^\lambda$$

$$V_\beta = \sum \frac{(c-b)(a-d)}{a+d}\delta_{c+b,a+d} f_c^\mu f_b^\nu \bar{f}_{a\mu}\bar{f}_{d\nu}$$

$$V_\gamma = \frac{2}{3}\sum (a-b-d)\delta_{c,a+b+d} f_c^\mu \bar{f}_{a\mu}\bar{f}_b^\nu \bar{f}_{d\nu}$$

Non-local deformations

- contribution from measure:

$$S_{\text{measure}} = 4\lambda^2 \sum f_n \bar{f}_n$$

- include quadratic tachyon

$$S_{\text{tachyon}} = \oint T(X) = a + u \sum f_n \bar{f}_n$$

Non-local deformations

- contribution from measure:

$$S_{\text{measure}} = 4\lambda^2 \sum f_n \bar{f}_n$$

(tachyon forced into existence!)

- include quadratic tachyon

$$S_{\text{tachyon}} = \oint T(X) = a + u \sum f_n \bar{f}_n$$

RG flow

λ $\rightarrow$ non-local deformation (keep symmetry)

a, u $\rightarrow$ control tachyon condensation

β-functions

$$\beta_a = -a - u$$

$$\beta_u = -u(1 - 8\lambda^2)$$

$$\beta_\lambda = -\frac{47}{6}\lambda^3$$

- λ increases
- tachyon condensation is modified
- obvious endpoint: $a = \infty$, $u = \infty$ (but there is more...)

Outline

Condensation of the 3-brane

- non-vanishing tachyon makes 3-brane unstable
- can we identify a process D3 $\to$ D2?
- assume a tachyon interaction

$$\oint \rho\left(X^2 - c^2\right)^2 = \rho c^4 - 4\rho c^2 \sum f_n \bar{f}_n + \mathcal{O}(f^4)$$

- ρ, c^2 are the 'natural couplings', in which a condensation to spherical brane is described
- re-write the β-functions:

$$\beta_{c^2} = 4 - 8\lambda^2$$
$$\beta_\rho = -\frac{\rho}{c^2}\left(c^2 - 16c^2\lambda^2 + 4\right)$$

Condensation of the 3-brane

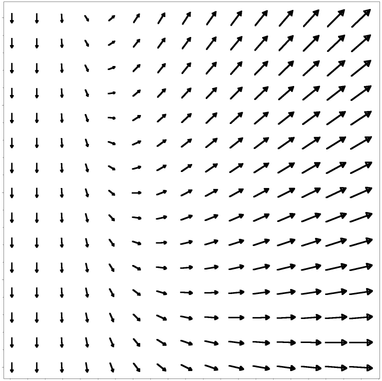
- for a 2-brane solution we would expect finite c^2 and running ρ *(λ fixed)*
- such a solution is

$$c^2 = \frac{1}{2}\lambda^{-2}$$
$$\beta_{c^2} = 0$$
$$\beta_\rho = -\rho(1 - 8\lambda^2)$$

- this tells us: ρ will increase
- geometry: determined by λ (curvature)
- $\lambda \to 0$ gives a flat brane *(flat space limit)*

λ–c^2-flow

- running of c^2 is strongly modified by the presence of λ



Verification of the solution

- Ansatz for D2-solution was a ‘guess’
- independent verification:
  - insert constraint $X^2 = c^2$ into action
  - verify conformality
- complicated $\longrightarrow$ look for stability check

$$A_1 = \frac{1}{2}\sum_{a=1}\sum_{d=2}\sum_{e=1}^{d-1}(d-a)\epsilon_{\alpha\beta} f_a^\alpha \bar{f}_{a+d}^\beta f_{e\gamma} f_{d-e}^\gamma$$

$$\bar{A}_1 = A_1^*$$

$$A_2 = \sum_{a=1}\sum_{c=1}\sum_{d=2}(d-a)\epsilon_{\alpha\beta} f_a^\alpha \bar{f}_{a+d}^\beta f_{c+d\gamma} \bar{f}_c^\gamma$$

$$\bar{A}_2 = A_2^*$$

$$A_3 = \frac{1}{4}\sum_{a=1}\sum_{b=1}\sum_{g=1}^{a+b-1}(a-b)\epsilon_{\alpha\beta} f_a^\alpha f_b^\beta \bar{f}_{g\gamma} \bar{f}_{a+b-g}^\gamma$$

$$\bar{A}_3 = A_3^*$$

$$A_4 = \frac{1}{2}\sum_{a=1}\sum_{b=1}\sum_{c=1}(a-b)\epsilon_{\alpha\beta} f_a^\alpha f_b^\beta f_{c\gamma} \bar{f}_{a+b+c}^\gamma$$

$$\bar{A}_4 = A_4^*$$

$$B_1 = \frac{1}{4}\sum_{m=2}\sum_{a=1}^{m-1}\sum_{b=1}^{m-1} m f_{a\alpha} f_{m-a}^\alpha \bar{f}_{b\beta} \bar{f}_{m-b}^\beta$$

$$B_2 = \sum_{m=2}\sum_{a=1}^{m-1}\sum_{b=1}^{m-1} m \bar{f}_{a\alpha} f_{m-a}^\alpha f_{b\beta} \bar{f}_{m-b}^\beta$$

$$B_3 = \frac{1}{2}\sum_{m=2}\sum_{a=1}^{m-1}\sum_{b=1}^{m-1} m f_{a\alpha} f_{m-a}^\alpha f_{b\beta} \bar{f}_{m-b}^\beta$$

$$C_1 = -\sum_{a,b,c,d=1}\frac{(a-b)(c-d)}{c+d} f_a^\alpha \bar{f}_{c\alpha} f_b^\beta \bar{f}_{d\beta}\delta_{a+b,c+d}$$

$$C_2 = \frac{1}{3}\sum_{a,b,c,d=1}(c-a-b)\bar{f}_a^\alpha \bar{f}_{b\alpha} \bar{f}_{c\beta} f_{a+b+c}^\beta$$

$$\bar{C}_2 = C_2^*.$$

Stability of the 2-brane

- consider case with vanishing tachyon
- will the tachyon be excited at higher loops?
- the tachyon counter-term:

$$\Sigma^{(2)}(p=0) = [\Lambda \ln \Lambda + (\gamma - 1)\Lambda] \left\{ \frac{4\lambda^2}{c^2 s^3} - \frac{8}{3}\frac{\lambda^2}{s^2 c^2} - \frac{38}{sc^4} \right\} + \ln \Lambda \{\text{terms} \propto u\} + \mathscr{O}(\lambda^4)$$

- all divergences vanish under a single condition:

$$c^2 = \alpha(s)\lambda^{-2}$$

- no ‘fine-tuning’ necessary

Interpretation

On the three-brane:

- λ excites infinitely many massive couplings
- affects stability of D-branes
- tachyon condensation process initiated and modified
- qualitatively new RG flows
- λ is running, c^2 can be kept finite, ρ is running & condensating
- breakdown of predictability $\rightarrow$ 2-brane Ansatz

On the two-brane:

- a priori tachyonic instability
- instability removed by essentially same condition on c^2 as on three-brane
- tachyon can be set to zero without fine-tuning
- indicates that 2-brane is conformal
- λ really seems to interpolate between flat and curved space $\rightarrow$ supports interpretation of λ as closed string perturbation

Perspectives

- higher order β-functions, more examples, better methods
- better control over flow
- other WZW-branes, potential
- backgrounds other than WZW
- supersymmetry
- systematic investigation of open-closed modulispaces, better statements on background independence
- boundary state formalism, possibly access to closed string vacuum
- connections to T-folds? Defect CFTs?

Thank You